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VOLUME XLIV NUMBER 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

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JULY 1916

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THE
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VOLUME XLIV JULY 1916 NUMBER 1

ANOMALOUS DISPERSION IN THE SUN. II

By SEBASTIAN ALBRECHT

In an earlier paper[1] with the foregoing title the writer has shown that lines in the solar spectrum which have close companions of sufficient intensity are displaced relative to wave-lengths obtained in the laboratory, and that these displacements are strikingly in accord, both in direction and in relative amounts, with the requirements of the anomalous-dispersion theory. The validity of these results and the methods by which they were derived have been brought into question in valued contributions by Dr. T. Royds[2] and by Director Evershed.[3] That the slight regard in which Evershed and Royds hold Rowland's Table of Solar Spectrum Wave-Lengths is not justified seems amply shown by the confirmation, by means of the Rowland tables, of the Mount Wilson pressure groups and pressure displacements. Further, when Dr. Royds questions the validity of the purely relative comparison between Rowland's solar wave-lengths, which are still the best that have been published, and the best laboratory wave-lengths of the iron arc, he assumes the responsibility of *proving* in what essential way his so-called direct-comparison methods are "more direct" for such a purely relative comparison of what may be termed "internal"

[1] *Astrophysical Journal*, 41, 333, 1915.

[2] *Kodaikanal Observatory Bulletin*, No. 48, 1915.

[3] *The Observatory*, 39, 59, 1916.

differences in the two systems. Systematic differences between the two systems as a whole or for any continuous section of spectrum are in no way involved. In either case the comparison is between two entirely distinct things, a solar spectrum and a laboratory spectrum. In the so-called direct method the advantage—which is of minor importance in this comparison—of having the two distinct spectra obtained nearly simultaneously and with the same instrument is in a measure counterbalanced by the disadvantage that the conditions under which the solar spectrum is taken are not necessarily the most favorable for the arc spectrum.

The principal object of this article is to offer a material addition to the evidence contained in the previous paper rather than indulge in a lengthy discussion of opinions. Before introducing the new material it may be well to recall the principal facts brought out in the former paper, p. 354:

> The evidence adduced above is quite definite as to the observed facts. These are: Fraunhofer lines, as given in Rowland's tables, are displaced when they have close companions; the displacement is (*a*) toward the violet when the companion is toward the red, and (*b*) toward the red when the companion is toward the violet. The displacement in (*a*) is greater than in (*b*), and in both it increases as the separation between the lines diminishes.

I wish next to quote from a recent contribution by Sir Joseph Larmor:

> Thus *very close spectrum lines ought to repel each other* to a degree that experiment alone can reveal.
>
> But this conclusion implies that the adjacent lines represent independent vibrations. If they are two components of a single compound vibration of the molecule, the argument is not applicable.
>
> If an iron line in the solar spectrum has a very close adjacent line *due to another substance*, while in the arc spectrum that substance is not present, then a displacement of the solar line relative to the arc line may be looked for.
>
> The displacement is—at any rate, on this theory—proportional to the density of the substance that is present and produces the adjacent disturbing line.[1]

In the original statement of the anomalous-dispersion theory Julius made no distinction, I believe, between pairs of lines in which the components are due to one element and those in which they are due to two different elements. In a more recent article[2]

[1] *The Observatory*, **39**, 103, 1916. [2] *Astrophysical Journal*, **43**, 49, 1916.

he says that the dispersion theory requires "the existence of a definite mutual influence of Fraunhofer lines separated from each other by very short distances, irrespective of the elements to which the lines are due"; and in a footnote, "provided those elements coexist in the mixtures at the same levels."

A provisional test for the distinction made by Larmor can be made from the data contained in my Tables V and VI (*op. cit.*, pp. 348 and 349). These are substantially reproduced in Tables I and II of the present article, with the modifications that the elements are indicated to which the companion lines are due, and the displacements[1] are inclosed in parentheses in the cases in which the companion lines are also due to iron. The lines thus differentiated are distributed among all the pressure groups.

As was pointed out in the former article (pp. 352 and 353), the results are not dependent upon the adopted system of weighting. However, since the displacement is distinctly a function of the separation of the line from its companion, and since the *certainty* of such displacement should be greater when the companion line is relatively strong, the weighted results deserve the preference. No proportionality between displacement and intensity was proposed, nor was there any intention to convey the impression that such proportionality exists. Nevertheless, the great multiplicity of very faint lines in the solar spectrum necessitates the tacit adoption of the principle that lower limits of intensity, and probably also of relative intensity, of the companion line exist, below which the line is incapable of producing the anomalous-dispersion effect here considered. With the moderate amount of material which is at present available it is not feasible to define, with great precision, such a lower limit. However, as can readily be verified by means of Tables I, V, and VI of the former article, moderately definite and, I believe, safe lower limits of intensity and of relative intensity of the companion line were adopted for the exclusion of lines for which the "certainty" of power to produce such an effect seemed too small. In view of the fact that it has been possible to determine a fairly satisfactory relation between separation and displacement,

[1] For this column I retain the heading "To Reduce Point to Curve" and the signs of the original tables. The term "displacements" as used in this article shall be irrespective of sign.

we may also hope for an actual determination, in the future, of approximate lower limits for the intensities. The essential requisite I believe to be a several-fold accumulation of data.

TABLE I

FE LINES WITH COMPANIONS TOWARD THE RED IN THE SOLAR SPECTRA

λ Rowland	Elements and Intensities (Companion: Line)	Separation	Weight	To Reduce Point to Curve	Group	Remarks
	Fe	A		A		
3705.708	2: 9	0.14	1	+.007	*a*1	
3735.014	Fe 4:40	0.47	½	(+.004)	*b*1	
3743.508	Ti 2: 6	0.12	1	+.006	*b*1	Also Fe—Burns
3834.364	Mn 4:10	0.14	1	+.009	*b*1	And 3:10, 0.36 to vi.
3997.547	2: 4	0.09	1	+.001	*b*4	And 1:4, 0.29 to vi.
4191.595	Fe 3: 6	0.25	1	(+.011)	*a* or *b*	
4204.101	La 4: 3	0.06	2	+.048	*b*3	And 2:3, 0.37 to vi.
4210.494	3: 4	0.07	3	+.027	*c*5	
4337.216	Cr 3: 5	0.51	½	+.002	*b*3	
4454.552	Ca, Zr 5: 3	0.40	1	+.003	*b*3	
4461.818	Fe–Mn 3: 4	0.36	1	+.008	*a*3	Provisionally retained on account of Mn
4476.185	Ag 3: 4	0.07	3	+.008	*b*4	
4489.911	Mn–Fe 3: 4	0.34	1	+.004	*a*3	Provisionally retained on account of Mn
4637.685	Fe 4: 5	0.51	½	(+.004)	*d*?	
4679.027	2: 6	0.38	1	+.005	*c*4	Ni? Also Fe (1)—Burns
4707.457	2: 5	0.22	1	(+.006)	*c*5	? Burns gives Fe 2 in arc
4727.582	Mn 2: 3	0.09	2	+.003	*d*?	
4938.997	Fe 2: 4	0.42	½	(+.005)	*c*	
4957.480	Fe 8: 5	0.30	2	(+.003)	*c*5	
4982.682	2: 4	0.31	1	+.013	*d*	
4985.432	Fe 3: 3	0.30	1	(+.009)	*c*	
5005.896	Fe 5: 4	0.41	1	(+.002)	*c*	
5012.252	1: 4	0.08	½	−.002	*a*	
5107.619	Fe 4: 4	0.20	2	(+.008)	*a*	
5125.300	Ni 1: 3	0.12	1	−.002	*d*	
5139.427	Fe 4: 4	0.22	2	(+.006)	*c*	
5195.113	Fe 2: 4	0.53	½	(+.006)	*a*	
5208.596	Fe 2: 5	0.18	0	+.003	.....	Cr line; weight 0—"group" is unknown
5273.339	Fe–Cr 2: 3	0.22	1	+.004	sub-*d*	Provisionally retained on account of Cr
5328.236	Cr 2: 8	0.28	½	−.007	*a*1	Double in sun?
5365.069	Fe 3: 5	0.53	½	(.000)	*e*	
5463.174	Fe 3: 3	0.32	1	(+.009)	*e*	
5476.500	Fe 3: 1	0.28	2	(+.011)	*a*	
5476.778	Ni 5: 3	0.34	2	+.015	*d*	And Fe 1:3, 0.28 to vi.
5598.524	Ca 4: 1	0.19	3	+.008	*e*	
6008.186	Fe 6: 4	0.60	½	(+.012)	*e*	
6078.710	Fe 2: 5	0.52	½	(−.002)	*e*	
6136.829	Fe 3: 8	0.38	1	(+.004)	*b*	
6400.217	Fe 2: 8	0.32	½	(+.001)	*d*	

TABLE I—*Continued*

	Summary*			Percentages	
	(1)	(2)	(3)	(3)÷(1)	(3)÷(2)
Weighted mean.....	+.0094	+.0115	+.0062	66	54
Sum of weights.....	45.5	27.5	18	.....	.....
Straight mean.....	+.0068†	+.0080	+.0055	81	69
Number of lines.....	38	20	18	.....	.....

* (1) = Results for all lines—(previously published).
(2) = Results when lines with Fe companions are omitted.
(3) = Results for the lines with Fe companions.

† In the original Table V (*op. cit.*) this was erroneously given as +.005.

The evidence summarized at the bottom of Tables I and II seems to show quite definitely that when the companion line is also due to iron the displacement is only half as great (average is 53 per cent) as when the companion is due to some other element. *This material reduction of the displacement for pairs in which both components are due to iron is distinctly in line with Larmor's theory.*

In Table III have been tabulated the ratios, companion toward violet: companion toward red, for the various subdivisions in the summaries of Tables I and II. It is seen that when the companion is toward the violet the displacement is quite closely two-thirds (or possibly five-eighths) as great as when the companion is toward the red. The marked approximation to uniformity in these ratios, whether we consider the weighted means or the straight means, was somewhat surprising. However, this is in part accidental.

Tables I and II of Royds (*op. cit.*) which were intended to prove my results to be "largely fictitious" were next examined from the point of view employed above. Of the 17 Fe lines with companion to the red, 10 (=60 per cent) of the companions are due to Fe and only 7 to other elements; of the 30 lines with companions to the violet, only 12 (=40 per cent) of the companions are due to Fe and 18 to other elements. While this disparity between the two sets of lines does not account for the smaller relative shift for the sun-arc displacements as compared with my residuals,[1] it does in part

[1] A satisfactory direct comparison between the sun-arc displacements of Evershed and my displacements does not seem feasible.

TABLE II

FE LINES WITH COMPANIONS TOWARD THE VIOLET IN THE SOLAR SPECTRUM

λ Rowland	Elements and Intensities (Companion: Line)	Separation	Weight	To Reduce Point to Curve	Group	Remarks
	Fe	A		A		
3647.988	Fe 4:12	0.43	½	(+.002)	*b*1	
3680.069	2: 9	0.25	½	−.007	*a*1	And 2:9, 0.46 to red
3722.729	Ni 3: 6	0.09	2	−.014	*a*1	Line also Ti?
3737.281	Ca–Mn 5:30	0.22	½	+.003	*a*1	
3746.058	Fe 8: 6	0.34	2	(−.008)	*a*1	
3748.650	Fe 10: 1	0.24	3	(−.008)	*b*1	
3887.196	Cr 3: 7	0.25	1	.000	*b*1	
3888.671	2: 5	0.11	1	−.008	*b*1	And Fe–Mn 2:5, 0.30 to red
3895.803	Mn 3: 7	0.22	1	.000	*a*1	
3969.413	6:10	0.53	½	−.004	*b*1	
4132.235	V 2:10	0.14	1	−.012	*b*1	Line is Fe–Co
4134.840	Fe? 3: 5	0.25	½	+.006	*b*4	Also V 1:5, 0.16 to vi. Weight ½ for V
4144.038	Fe, Mo 4:15	0.47	½	−.005	*b*1	Mo, 2:15. 0.37 to vi. Retained on account of Mo
4147.836	2: 4	0.33	1	+.002	*a* or *b*	
4191.843	Fe 6: 3	0.25	2	(+.003)	*e*	
4227.606	Ca 20: 4	0.70	½	−.004	*d*5	And 1:4. 0.13 on same side
4233.772	Mn 4: 6	0.44	½	+.002	*d*5	
4291.630	Ti 2: 2	0.35	1	+.007	*a*	And 1:2. 0.26 on same side
4294.301	Ti 2: 5	0.10	2	−.005	*b*2	
4308.081	Ca 3: 6	0.17	2	−.006	*b*1	And fainter line on each side
4315.262	Ti 3: 4	0.12	2	−.007	*b*3	
4407.871	V 2: 4	0.06	2	+.011	*c*4	Also V, 2:4, 0.49 to red
4427.482	Ti 2: 5	0.22	1	.000	*a*3	
4531.327	Fe?, Co 2: 5	0.20	½	−.002	*b*3	And 2:5, 0.47 to red
4556.306	Fe 3: 4	0.24	1	(+.002)	*e*	
4592.840	Ni 2: 4	0.13	2	−.010	*c*4?	
4668.331	2: 4	0.09	2	−.010	*d*?	
4727.676	Fe 3: 2	0.09	0	−.029	.....	Mn, weight 0—"group" unknown
4787.003	Ni 3: 2	0.28	1	−.012	*c*4	
4789.849	Cr 2: 3	0.32	1	−.012	*c*4	
4872.332	1: 4	0.22	½	−.006	*c*5	
4878.407	Ca 3: 4	0.09	2	−.008	*c*5	
4939.868	Fe 2: 3	0.45	½	(−.002)	*a*	
4957.785	Fe 5: 8	0.30	1	(−.004)	*c*5	
4985.730	Fe 3: 3	0.30	1	(+.003)	*c*	
5006.306	Fe 4: 5	0.41	1	(−.002)	*c*	
5028.308	Fe 1: 2	0.37	½	(.000)	*a*	Companion not in Burns's Fe table
5041.255	Fe 3: 4	0.19	1	(−.002)	*a*	
5041.936	Ca 2: 4	0.14	1	+.001	*a*	
5079.409	Fe 3: 4	0.25	½	(−.006)	*a*	
5098.885	Fe 1: 3	0.13	½	(−.005)	*a*	
5107.823	Fe 4: 4	0.20	2	(−.004)	*a*	

TABLE II—*Continued*

λ Rowland	Elements and Intensities (Companion: Line)	Separation	Weight	To Reduce Point to Curve	Group	Remarks
	Fe			A		
5139.644	Fe 4: 4	0.22	2	(+.002)	*c*	
5167.678	Mg 15: 5	0.18	3	−.016	*a*	
5195.647	Fe 4: 2	0.53	½	(−.001)	*e*	
5202.516	Fe? 2: 4	0.08	1	−.006	*a*	Companion not in Burns's Fe table. Weight reduced
5208.776	Cr 5: 2	0.18	3	−.006	sub-*d*	
5227.362	Fe–Cr 3: 5	0.32	1	−.003	*a*4	Provisionally retained on account of Cr
5270.558	Ca 3: 4	0.12	2	−.026	*a*4	
5273.558	Fe 3: 2	0.22	2	(−.014)	sub-*d*	
5333.089	Co, Fe 1: 4	0.24	½	+.005	*a*4	Provisionally retained on account of Co
5365.596	Fe 5: 3	0.53	½	(+.003)	*a*	
5371.734	Cr? 4: 3	0.08	3	−.042	*a*1	
5404.357	Fe 2: 5	0.33	1	(−.012)	*e*	
5447.130	Ti 2: 6	0.33	½	+.003	*a*	Double in sum?
5455.834	Fe? 2: 4	0.16	2	(−.007)	*a*	Fe—Burns
5463.494	Fe 3: 3	0.32	1	(−.002)	*e*	
5594.884	Ca 4: 1	0.19	2	+.009	*e*	Measured only by Burns with large probable error.
5603.186	Ca 3: 4	0.10	2	−.011	sub-*d*	
5615.877	Fe 2: 6	0.36	1	(−.004)	sub-*d*	
5659.052	Fe 1: 4	0.30	½	(−.003)	sub-*d*	? Also Cr 0:4, 0.17 to vi
6020.401	H, Fe? 2: 4	0.17	1	−.008	*b*?	
6148.040	2: 3	0.09	2	−.005	*d*	
6191.779	Ni 6: 9	0.39	1	−.004	*b*	
6254.456	1: 5	0.07	½	+.017	*b*	Doubtful

	Summary *			Percentages	
	(1′)	(2′)	(3′)	(3′)÷(1′)	(3′)÷(2′)
Weighted mean	−.0066	−.0077	−.0040	61	52
Sum of weights	81.5	53.0	27		
Straight mean	−.0041	−.0047	−.0030	73	64
Number of lines	64	41	23		

* (1′) = Results for all lines previously published.
(2′) = Results when lines with Fe companions are omitted.
(3′) = Results for the line with Fe companions.

explain why the displacement for companion to the violet is not smaller than for companion to the red, as would be required by the anomalous-dispersion theory.

TABLE III

RATIOS: COMPANION TOWARD VIOLET ÷ COMPANION TOWARD RED

	(1')/(1)	(2')/(2)	(3')/(3)	Mean of Three Ratios
For weighted means.....	0.70	0.67	0.65	**0.67**
For straight means.....	0.60	0.59	0.55	0.58
Average.....	(0.65)	(0.63)	(0.60)	(0.62)

As the displacement has not entirely disappeared for pairs in which both lines are due to iron, we must conclude that the components of these pairs represent only in part actual physical connection in the molecule, and in part entirely independent vibrations. As was pointed out by Larmor (*op. cit.*) concerning two adjacent lines of the *same* substance:

> If these lines are in physical connection in the molecule, as components of one compound mode of vibration, no sensible difference would be expected, as regards their separation, between the solar spectrum and the arc spectrum; if they are independent lines, then they should separate from one another as the density of the substance that produced them increases.

In order that lines of the *same* substance may be independent, the most natural inference is that they originate at different levels in the solar atmosphere. The actual way in which lines may depend upon level cannot be determined here. We can, however, make a provisional—though far from satisfactory—test of our problem on the hypothesis, elaborated by St. John,[1] "that the lines of any one element originate at depths increasing with decrease of solar intensity, that the enhanced lines are higher than unenhanced lines of equal solar intensity, and that we see into the sun to greater depth at the red end of the spectrum than at the violet." With this as a working hypothesis, the components of a close pair of lines of equal intensity would have their origin at the same effective solar level and thus have a common origin, representing, at least in a large measure, two components of a single compound vibration of the molecule; lines differing in intensity are supposed to originate at different levels in the solar atmosphere, i.e., in

[1] *Astrophysical Journal*, **37**, 377, 1913; **38**, 157, 1913; **40**, 45, 1914; *Mt. Wilson Contr.*, Nos. 69, 74, and 88.

separate and distinct masses of the same gas. In the latter case the components of a pair would have no connection within the molecule, and, for the purposes of our problem, should be as independent of each other as though they were due to two different elements. This independence would necessarily be complete only in so far as there is a complete separation of the respective masses of gas in which the two components originate.

TABLE IV

DISPLACEMENTS, FOR COMPANION TOWARD THE RED, ARRANGED ACCORDING TO RATIO OF INTENSITIES, FE COMPANION : FE LINE

λ	Ratio of Intensities	Separation	Weight	To Reduce Point to Curve	Straight Means	No. of Lines	Weighted Means	Sum of Weights	Mean Separation	Mean Ratio	Weighted Means for Separation of 0.26A
		A		A	A		A		A		A
4985.4	3: 3	0.30	1	+.009							
5403.1	3: 3	0.32	1	+.009	+.0080	4	+.0077	6	0.26	1:1	+.0077
5107.6	4: 4	0.20	2	+.008							
5139.4	4: 4	0.22	2	+.006							
4637.6	4: 5	0.51	½	+.004							
5365.0	3: 5	0.53	½	±.000							
4938.9	2. 4	0.42	½	+.005							
5195.1	2: 4	0.53	½	+.006	+.0040	6	+.0050	3½	0.46	0.6:1	+.0086
4191.5	3: 6	0.25	1	+.011							
6078.7*	2: 5	0.52	½	−.002							
4707.4	2: 5	0.22	1	+.006							
6136.8	3: 8	0.38	1	+.004	+.0038	4	+.0042	3	0.35	0.3:1	+.0058
6400.2	2: 8	0.32	½	+.001							
3735.0	4:40	0.47	½	+.004							
5005.8	5: 4	0.41	1	+.002							
6008.1	6: 4	0.60	½	+.012	+.0070	4	+.0064	5½	0.40	1.8:1	+.0089
4957.4	8: 5	0.30	2	+.003							
5476.5	3: 1	0.28	2	+.011							

* Grouped with lines of higher ratio because companion is enhanced.

Accordingly, the pairs of lines in which both components are due to iron were selected from Tables I and II and arranged in the order of their ratios of intensities as shown in Tables IV and V. The order is that of progressively increasing difference from equality in the intensities of the two components. It was deemed desirable to keep separate the small number of lines for which the ratio of intensities is greater than unity, and these were all placed into one group at the bottom in each table.

Since the displacement is a function of the separation of the line from its companion the means in Tables IV and V require a reduction to a uniform separation before they can be directly

compared. The curves *a* and *b* in the former article, Fig. 2, p. 353, are represented sufficiently well by the linear equations

$$y_a = -0.0182x_a + 0.0124$$
$$y_b = -0.0173x_b + 0.0087$$

These two lines are nearly parallel, and we may adopt a mean value of -0.018 for the slope. This holds for the complete data

TABLE V

DISPLACEMENTS, FOR COMPANION TOWARD THE VIOLET, ARRANGED ACCORDING TO RATIO OF INTENSITIES, FE COMPANION: FE LINE

λ	Ratio of Intensities	Separation	Weight	To Reduce Point to Curve	Straight Means	No. of Lines	Weighted Means	Sum of Weights	Mean Separation	Mean Ratio	Weighted Means for Separation of 0.26A
		A		A	A		A		A		A
4985.7	3: 3	0.30	1	+.003							
5463.4	3: 3	0.32	1	−.002							
5107.8	4: 4	0.20	2	−.004	+.0002	5	−.0001	7	0.26	1:1	−.0001
5139.6	4: 4	0.22	2	+.002							
4556.3*	3: 4	0.24	1	+.002							
5006.3	4: 5	0.41	1	−.002							
5041.2	3: 4	0.19	1	−.002	−.0033	3	−.0028	2½	0.28	0.8:1	−.0032
5079.4	3: 4	0.25	½	−.006							
4939.8	2: 3	0.45	½	−.002							
4957.7	5: 8	0.30	1	−.004	−.0032	4	−.0048	4	0.32	0.6:1	−.0059
5028.3	1: 2	0.37	½	±.000							
5455.8	2: 4	0.16	2	−.007							
3404.3	2: 5	0.33	1	−.012							
5098.8	1: 3	0.13	½	−.005							
5615.8	2: 6	0.36	1	−.004	−.0044	5	−.0054	3½	0.31	0.3:1	−.0063
3647.9	4:12	0.43	½	+.002							
5659.0	1: 4	0.30	½	−.003							
3746.0	8: 6	0.34	2	−.008							
5273.5	3: 2	0.22	2	−.014							
5365.5	5: 3	0.53	½	+.003	−.0042	6	−.0061	10	0.35	3.1:1	−.0077
5195.6	4: 2	0.53	½	−.001							
4191.8	6: 3	0.25	2	+.003							
3748.6	10: 1	0.24	3	−.008							

* Grouped with lines of higher ratio because companion in enhanced.

of Tables I and II of the present article. What the correct slope may be for the corresponding curves for Tables IV and V cannot be determined satisfactorily from the data there given. Qualitatively the results will not be seriously affected by adopting the slope found above. The last column in Tables IV and V gives the weighted means of column 7[1] reduced to the uniform separation of 0.26A with the use of slope 0.018.

[1] The reasons for giving preference to the weighted results were stated above. The straight means give substantially the same results.

The evidence thus furnished is not as definite as could be desired. Table IV indicates practically no relation between displacement and difference in intensity of the components of close pairs, while Table V seems to show zero displacement for equality in the intensities of the two component lines and a progressive increase of displacement with increasing difference in the intensities. Whether this lack of accord between the two cases be due to insufficient data alone, or whether St. John's theory represents only imperfectly the actual relation between solar levels and intensities, must be left in abeyance. The principal difficulty which the writer experiences for a complete acceptance of St. John's theory is the requirement that the lines separate in the different solar levels strictly according to intensity, whereas we are accustomed to expect marked differences in intensity in spectra from very limited and presumably homogeneous portions of light-sources in the laboratory. The unqualified acceptance of St. John's theory would seem to require the conclusion that it is impossible (or, at least, it has been impossible up to the present) to obtain substantially homogeneous conditions in a very limited portion of the arc, or spark, or electric furnace. Or, are we to conclude that the complete significance of St. John's observed facts is still obscure? For the present, therefore, the moderate preponderance of evidence in favor of a relation between displacement and ratio of solar intensities is to be considered as possibly a crude indication of such a relation, though even this is to be taken with reserve.

In conclusion it may be well to recall that results similar to those published in my former article had been obtained[1] from a much larger number of lines discussed entirely without reference to pressure-effect. This justifies the belief that the introduction into the problem of density-effect and pole-effect—and these should be taken into account as soon as they have entered upon a sufficiently advanced stage of development—will leave the evidence adduced above qualitatively intact, while it may alter it quantitatively. In fact, these effects have possibly been taken into account indirectly. According to Royds, pressure-shifts and shifts due to differences in vapor-density are somewhat interlaced; with

[1] *Op. cit.*, p. 335, n. 2.

increased atmospheric pressure, he believes there will also be increased vapor-density in the arc. If this reasoning be applied to the Mount Wilson pressure displacements—and it should probably be extended to apply also to conditions in the sun—then not only true pressure-shifts but in part shifts due to vapor-density have been indirectly taken care of in my reductions.

Naturally, no position taken in regard to the interpretation of observed facts need be irrevocable. While the evidence originally adduced is still substantially in favor of anomalous dispersion in the sun, and while it has been strengthened by the separation of the pairs according as the components are due to one element or to two different elements, there is nevertheless no immediate necessity for making a final decision one way or the other. The reopening of the problem for dicussion by several investigators and from new points of view is certainly giving promise to ample rewards, both for the main problem and for other problems which are indirectly involved.

SUMMARY

In a former article it was shown that iron lines with close companions in the solar spectrum (Rowland's Preliminary Table of Solar Spectrum Wave-Lengths) are displaced relative to their positions in the arc spectrum; when the companion is to the violet the displacement is toward the red, and when the companion is to the red the displacement is toward the violet; in the former case the displacement is only two-thirds as great as in the latter, and in both cases it diminishes progressively with increasing separation of the two lines. These observed facts are strikingly in accord with the requirements of the anomalous-dispersion theory of Julius. Personality in the measurement of close pairs of lines was also suggested as a possible cause for the observed facts. However, it would be difficult to explain, on this ground, the observed inequality of the displacements for the two components.

In the present investigation a marked distinction was found to exist between pairs of lines in which both components are due to iron, and those in which one of the components is due to some other element. In the latter case, both for companion to the red and for companion to the violet, the displacement is only one-half as

great as in the former case. As above, so also in each of these subdivisions, the displacement for companion to the violet is only two-thirds as great as for companion to the red. On the anomalous-dispersion theory the observed smaller displacement for pairs of lines in which both components are due to iron is explained on the basis that the components of these pairs represent only in part physical connection in the molecule and in part entirely independent vibrations. These observed facts are also in accord with the anomalous-dispersion theory as modified by a recent suggestion of Sir Joseph Larmor.

For the lines with Fe companions an attempt was made to determine a relation between displacement and difference in solar level between the line and its companion, by utilizing the hypothesis, elaborated by St. John, "that the lines of any one element originate at depths increasing with decrease of solar intensity." The moderate preponderance of evidence in favor of a relation between displacement and ratio of intensities of the two components is not sufficient to be regarded as more than possibly a crude indication of such an effect.

The observed facts outlined in paragraphs one and two of the summary seem sufficiently definite to be considered established. In regard to their interpretation it may be said that the theory of anomalous dispersion in the sun as developed by Julius and modified by Larmor does account for them. However, as this subject is quite clearly still in the early stages of development, final judgment may well be suspended for the present.

DUDLEY OBSERVATORY, ALBANY, N.Y.
May 5, 1916

NOTE

While the manuscript for the above article was in the hands of the printer an interesting communication appeared in which St. John shows,[1] by two lines of evidence, that "no factor of transformation nor any curve can yield true differences between the Rowland and international wave-lengths for all lines even for a limited spectral region." A careful perusal of my first paper on anomalous dispersion in the sun will show that I am entirely in accord with this

[1] *Proceedings of the National Academy of Sciences*, **2**, 226, 1916.

conclusion. However, as my two articles rest upon the basis of a comparison between the Rowland and international wave-lengths, and in order to avoid an erroneous inference—which might easily be possible—it was deemed advisable to call special attention to the fact that no such direct comparison of the two systems of wave-lengths is involved in my work. A more accurate reference to my method of comparing the two systems would be: "Albrecht uses as ordinates the quantities, Rowland minus *corrected international*, and from the mean curve derives directly the systematic difference for *a very much restricted set* of lines." In fact, my method of comparison was devised because I was fully aware of both the pressure-effect and the effect according to solar intensity, and because I wished the method to be sufficiently flexible to allow of future correction for other possible effects which at present are entirely unknown.

July 10, 1916

THE ACCURACY OBTAINABLE IN THE MEASURED SEPARATION OF CLOSE SOLAR LINES; SYSTEMATIC ERRORS IN THE ROWLAND TABLE FOR SUCH LINES[1]

By CHARLES E. ST. JOHN and L. W. WARE

1. INTRODUCTION

Close pairs of solar lines play such a rôle in the search for evidence of mutual influence between neighboring lines in the solar spectrum—a deduction from the anomalous-dispersion hypothesis—that it is important to have an idea of the accuracy obtainable in their measurement, and hence of the relation that the errors bear to the probable magnitude of the suggested effect. The investigation thus necessitated is also preliminary to a determination of the wave-lengths of the solar lines in international units, and in particular is an effort to develop a method of measuring the wave-lengths of such as have lines closely adjacent.

In investigations on the resolving power of spectroscopes, a complex line is considered to be resolved when the intensity at the midpoint of the overlap between two lines of equal intensity is 0.81 of the maximum for the single lines. For precision in the measurement of close doubles in the solar spectrum the important consideration is the lower limit for the difference in wave-length between lines of a specified intensity and character, that can be measured to a given accuracy with the spectrographs and photographic plates now available, rather than the resolving power which shows the line as double. Though the above standard of resolution is sufficient for determining the complex character of a solar doublet, the accuracy of the wave-length determination for the separation implied by it has not previously been investigated. Moreover, since the conditions presented by the unequal intensity of the background on the two sides of each component are such as may introduce systematic personal errors, it has seemed desirable to examine this question

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 120.

in detail; and, in fact, the results of the examination form an important portion of this contribution.

2. APPARATUS

For the present investigation an excellent grating by Anderson has been at our disposal through the kindness of the Physical Laboratory of the Johns Hopkins University. The ruled surface is 114×160 mm and the number of lines is 95,000. The definition is all that can be asked, even when the whole surface is used. As determined from the iodine absorption lines, the excellent performance is retained up to and including the fifth order. It has been used as a Littrow spectrograph, focus 30 feet (9.14 m), in connection with the 60-foot tower telescope. For comparing the results given by large-scale spectrograms obtained by long focus and high resolving power, respectively, a series of plates was taken with a large grating by Michelson, number of lines 77,500, used in the 75-foot spectrograph of the 150-foot tower equipment. The slit-width was usually four-normal, but for the very difficult separations a two-normal slit was employed, giving 94.3 per cent of the resolution for an indefinitely narrow slit[1] with monochromatic light. Commercial photographic plates have been used as follows: for the blue-violet, Seed's Process plates; for the green, Cramer's X-ray for the most part, but in special cases where fineness of grain was particularly important, Process plates sensitized with pinaverdol; for the yellow-red, Wratten "M" plates and Seed's Process sensitized according to the Wallace formula.[2] Backed plates have been used with advantage for some of the most difficult separations.

Upon a plate of an intensity suitable for the general run of lines, the measurement of the separation and, hence, of the wave-lengths of the components of a close pair, in which one or both components are strong lines, is difficult; but with exposure and development adapted to the case, somewhat too strong for weak lines, the difficulties and uncertainties are decreased with a corresponding in-

[1] Schuster, *Astrophysical Journal*, **21**, 207, 1905; Zeeman, *K. Akad. Amsterdam Proceedings*, **18**, 412–415, 1915.

[2] *Astrophysical Journal*, **26**, 317, 1907.

crease of confidence in the results. The exposure times and the intensity of development have been chosen to obtain the best conditions for the particular separation under consideration.

For measuring the spectrograms the usual filar-micrometer instruments have been employed; in addition a registering microphotometer of the Koch form has been used for purposes of checking, and also for some absolute determinations where it appeared especially difficult to eliminate the personal equation.

3. METHODS OF MEASUREMENT

In the measurement of the separation of close pairs of solar lines some means of determining whether the results, barring accidental errors, represent the real separation or are affected by systematic errors is an important desideratum. That concordant values are obtained by repeated measurements of the same or of other similar plates by a single observer has little bearing upon the question, as possible systematic errors introduced by the personal equation of the observer are not thereby eliminated. This comes into play in estimating the position of the maximum intensity of a line, one of whose edges is seen against a strong continuous spectrum background, and the other against a background—the region of overlap—whose intensity may differ but little from that of the line itself. In order to obtain a check upon the measurements for ordinarily close pairs, all determinations have been made by at least two observers, and in order to vary the conditions widely, they have been made upon a series of six spectrograms representing the first five orders of the 30-foot and the first order of the 75-foot spectrograph. The appearance of a close pair of lines differs greatly under these conditions, as the scale of the spectrograms ranges from 1.8 to 0.25 A per mm and the resolving power changes by fivefold. Under such widely different circumstances any personal error depending upon the proximity and character of the adjacent lines will not remain constant, and concordance among the results can be taken to indicate the practical elimination of such error. For selecting the data to be used in forming the final mean separation for a given pair of lines the criterion is, therefore, the degree of concordance between the orders. Beginning with

the highest, those orders are included for which the agreement between the orders is comparable to that between the measurements within the orders.

For pairs which with the highest resolving power and the largest scale employed are not completely separated, but yet appear accurately measurable with the filar micrometer, curves have been drawn by the registering microphotometer. Separations so obtained appear to be least affected by errors, either systematic or accidental, and to represent most nearly the true separation of the components.

For doublets at or very near the limit of resolution, whose components cannot be sufficiently separated for micrometric settings upon their maxima to be made without influence by the neighboring line, and are not reproduced as discrete curves by the registering microphotometer, the following method has been used to obtain the separation and furnish a check upon the attempted micrometer measurements. In such cases it is generally possible to make intensity classifications for the components as precisely as for the free-standing lines. An inspection of the plate enables one to select the free-standing lines that correspond most nearly to the components of the complex line. Microphotometer curves are obtained for the complex line and the selected list of isolated lines, the sensitiveness of the instrument being held constant during the run over the plate. From the curves of the isolated lines one selects those that conform to the violet and red branches of the compound curve, superposes the negatives so that the corresponding branches of the three curves are coincident, and measures the distance between the axes of the constituent curves. The scale of the spectrograms used for this purpose, 1 mm = 0.30 A, is increased fifty-fold by the microphotometer, so that the scale of the curves is 1 mm = 0.006 A. The measurements are made with sufficient accuracy with a fifth-millimeter scale. The operation is an analysis of the doublet, since one determines not only the separation but also the form and intensity of the components.

A less satisfactory procedure is to measure upon the original spectrogram the widths of a large number of isolated lines of the same nominal intensities as the components of the doublet by

setting upon their edges as in the measurement of star-images. Similar settings are made upon the free edges of the complex line. Its width decreased by the half-widths of the two sets of free-standing lines furnishes a measure of the separation of the components. The weak point of the method lies in the fact that lines of the same nominal intensity vary greatly, even in the same spectral region, as shown by the curves of widely different heights and characters which they yield; but by including several reference lines of each intensity the results are in fair agreement with those given by the microphotometer.

4. ILLUSTRATIONS OF THE METHODS

a) ***Filar micrometer.***—A number of doublets in the violet region were measured for separations upon a fifth-order plate of excellent definition, scale 1 mm=0.29 A. One exposure was measured by an observer five times at intervals extending over two months; three exposures were measured by another observer, once each. Both observers had had experience in the measurement of solar lines. The results are shown in Table I. In the first column between the Rowland wave-lengths, an indication is given of the appearance of the doublets. The symbol $s<<<<c$ means that the space between the maxima is very much less dark than the continuous background bounding the free edges of the components, i.e., the doublet is barely resolved; while for the pair at λ 4457, $s=c$, there is practically complete resolution. In the eighth and ninth columns are the means of the series of 5 and 3 measures, and in the tenth column the differences between the Rowland and the mean Mount Wilson separation. These differences are all positive and indicate an average systematic error of 0.006 A, either in the Rowland tables or in the Mount Wilson measures, carrying with it a corresponding error in the wave-lengths of the constituent lines. In the last column are given the separations obtained by the same observers upon a third-order spectrogram of excellent quality. The agreement with the fifth-order plate is mutually corroborative. The systematic character of these discrepancies between the Rowland and Mount Wilson results lends them a greater significance than their magnitude alone would imply. Further evidence of

their reality and the reasons for considering them actual errors in the Rowland values will be given in later sections of the paper.

b) *Registering microphotometer.*—The Mg-Fe pair at λ 5167 and the Fe pair at λ 5169 furnish examples of the unrecognized difficulty and the unsuspected systematic errors that occur in filar-micrometer determinations of separations apparently within the

TABLE I

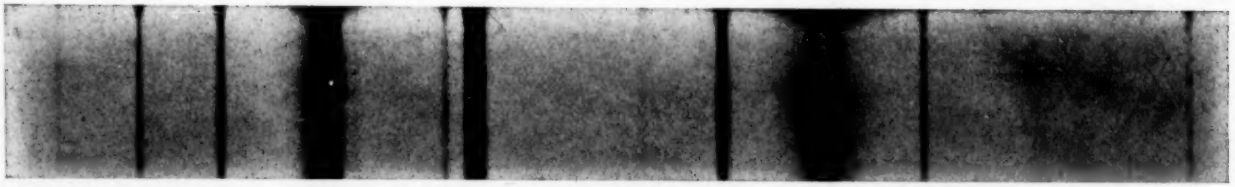
MICROMETRIC MEASURES ON CLOSE PAIRS OF SOLAR LINES

Pairs	Int.	Order V							Rowland *minus* Mt. Wilson	Order III
		1	2	3	4	5	S (5)	W (3)		
4455.980	2									
s<<<<*c*	..	0.067	0.078	0.072	0.072	0.075	0.073	0.078	+0.009	0.074
.064	3									
4457.600	2									
s=*c*	..	.108	.107	.105	.106	.104	.106	.106	+ .006	.108
.712	2									
4459.199	2									
s<<<*c*	..	.093	.098	.093	.094	.096	.095	.093	+ .008	.091
.801	3									
4464.844	2									
s<*c*	..	.088	.086	.088	.088	.088	.088	.090	+ .005	.091
.938	1									
4472.884	1									
s<<<*c*	..	.079	.080	.068	.078	.082	.077	.082	+ .004	.076
.967	0									
4476.185	4									
s<<<<*c*	..	.071	.065	.067	.064	.064	.066	.068	+ .001	.070
.253	3									
4482.338	5									
s<<<<*c*	..	0.091	0.091	0.089	0.096	0.092	0.092	0.097	+0.006	0.094
.438	3									
Mean.....	.....	.....	.....	.....	.....	.....	.....	.....	+0.006	.....

range of ordinary methods. In these cases the resolution under high power is so nearly complete that it has not been easy to convince one's self that the micrometer settings upon the components are subject to the systematic errors that appear when the results are compared with those obtained from the registered curves. In Plate I (*a*) a spectrum of the fourth order is reproduced, magni-

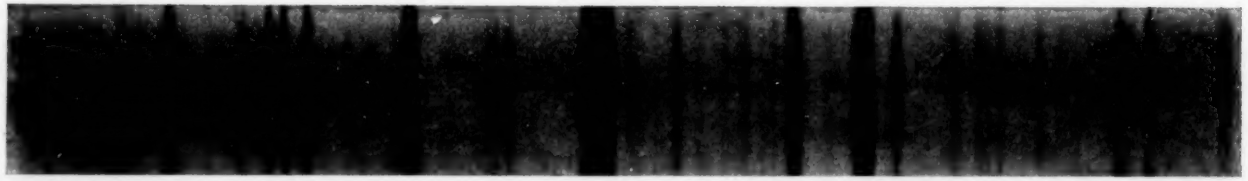
PLATE I

a



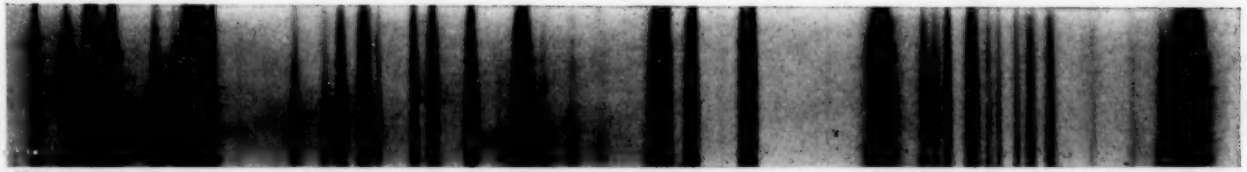
λ5167 λ5169 Fourth Order. Enlargement 4.7

b



λ5107 Fourth Order. Enlargement 4.7

c



λ3918 λ3919 Fourth Order. Enlargement 4.7

Separation: Rowland 0.099 0.101
Mount Wilson 0.099 0.093

fication 4.7, and in Figs. 1 and 2 are the corresponding curves drawn by the registering photometer.

Under the title "An Adaptation of the Koch Registering Microphotometer to the Measurement of the Sharpness of Photographic Images"[1] Tugman shows that the upper slit of the microphotometer

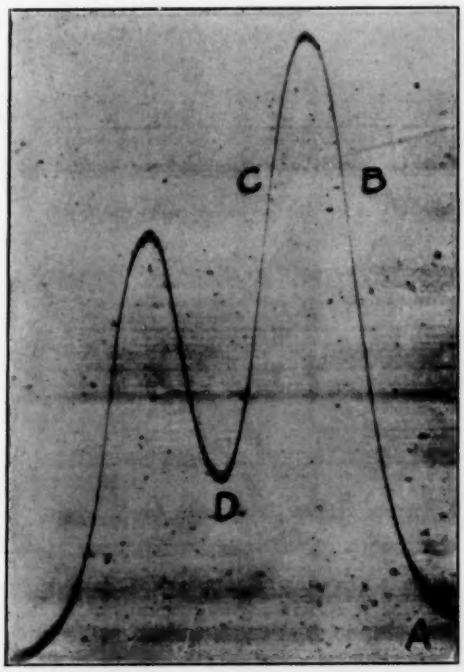


FIG. 1.—Microphotometer curve of Mg–Fe pair λ 5167.

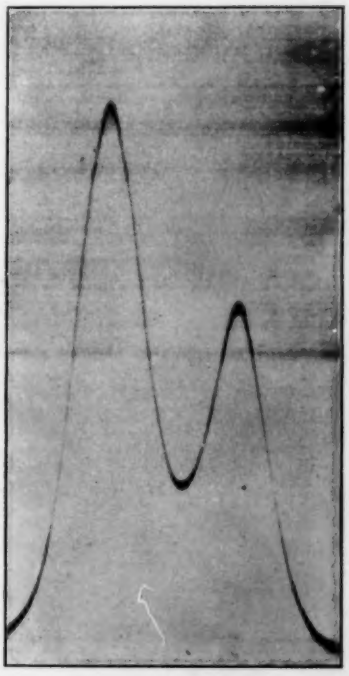


FIG. 2.—Microphotometer curve of Fe pair λ 5169.

should not cover more than one-fifth of the total width between maximum and minimum density, i.e., the half-width of the spectrum line; but that with slit-widths less than 0.75 mm the pinhole effect begins to enter, which falsifies the density-gradient by widening the base of the curve. Such an effect, while distorting the curve and lowering the resolution by the basal widening, does not shift the axis of the curve for an isolated line; and the free portions

[1] *Astrophysical Journal*, 42, 321, 1915.

of the components of a nearly separated doublet, such as the pair at λ 5167 or at λ 5169, appear to be unaffected.

The shortest distance between maximum and minimum intensity occurs between the lowest point in the region of overlap and the peak of the smaller curve. Upon the high-dispersion spectrogram used the horizontal distance is 200 μ for λ 5167. The objective magnifies fivefold; with a slit-width of 100 μ one-tenth of the image of the half-line is covered. The symmetrical character of the free portions of the two curves strikes one at a glance. The distance between their axes is 430 μ=0.158 A. Three observers with long experience in the measurement of solar lines by the filar micrometer found separations of 0.175, 0.169, and 0.172 A, respectively. That of the Rowland table is 0.181 A.

Third- and fifth-order spectrograms yield separations consistent with those of the fourth order, showing similar discrepancies between micrometer measures and the values derived from the registered curves. If the Rowland value represents the true separation, there must be a relative instrumental displacement of the axes of the curves of 3 to 4 mm, which seems inadmissible. The two methods show similar results for the pair at λ 5169, namely, 0.122, 0.136, and 0.151 A for the registering microphotometer, filar micrometer, and Rowland, respectively. To ascribe such discrepancies to systematic errors in the filar-micrometer settings, as seems the probable explanation, is a decided shock to all the observers, since they thought in advance that the settings could be made without systematic errors. The resolution upon the original spectrogram is more nearly complete than the curves indicate, as the pinhole effect has accentuated the overlap.

c) Analysis.—For pairs with components of intensities 3 and 4 on the Rowland scale the limit of resolution measurable by micrometer appears to be in the neighborhood of 0.10 to 0.15 A, increasing with the wave-length. In Table II are given the separations for very close pairs determined by micrometric settings upon (1) the maxima of the components, (2) the edges of the doublets and the reference lines, and (3) by analysis of the curves registered by the microphotometer. The spectrograms were taken in the fifth order, the scale varying from 0.23 to 0.30 A per

millimeter in passing from the green to the violet. The seventh column in Table II contains the unweighted means of the three methods of determination, and the last the Rowland *minus* Mount Wilson differences, which again are all positive with a

TABLE II

COMPARISON OF METHODS FOR THE SEPARATION OF PAIRS AT THE LIMIT OF RESOLUTION

ROWLAND			FILAR MICROMETER		ANALYSIS	MEAN Δλ	R.—MT. W.
Wave-Lengths	Int.	Δλ	On Max.	On Edges			
4204.101 s<<<<<<c .163	3 4	0.062	0.046	0.040	0.041	0.042	+0.020
4219.516 s<<<<<<c .580	4 3	.064	.053	.045	.049	.049	+ .015
4476.185 s<<<<<c .253	4 3	.068	.068	.063	.068	.066	+ .002
4482.338 s<<<<<c .438	5 3	.100	.094	.081	.087	.087	+ .013
4878.313 s<<c .407	3 4	.094	.093	.086	.086	.088	+ .006
5264.329 s<<<<<c .415	4 3	.086	.079	.068	0.75	.074	+ .012
5270.438 s<<<<c .558	3 4	0.120	0.104	0.108	0.098	0.103	+0.017
Mean R.— Mt. W.....	.....	.....	.....	.....	.....	.....	+0.012 A

mean magnitude of 0.012 A. The registered curve for the pair at λ 4219, Rowland intensities 3 and 4, is reproduced in Fig. 3, with curves of the isolated lines λ 4201, intensity 3, coincident with the red branch, and λ 4196, intensity 4, coincident with the violet branch. The separation derived from these component curves is 0.051 A, the Rowland value being 0.064 A. Although the gradation-curve corresponding to the free edge of the doublet does not

represent the actual distribution of energy in the line, it does represent the slope and height of a curve due to a free-standing line which in intensity and character is identical with the corresponding component of the doublet. Any errors, due to the pinhole effect, in reproducing the density-gradient of the free edges of the components occur also in the curves of the isolated lines of the same

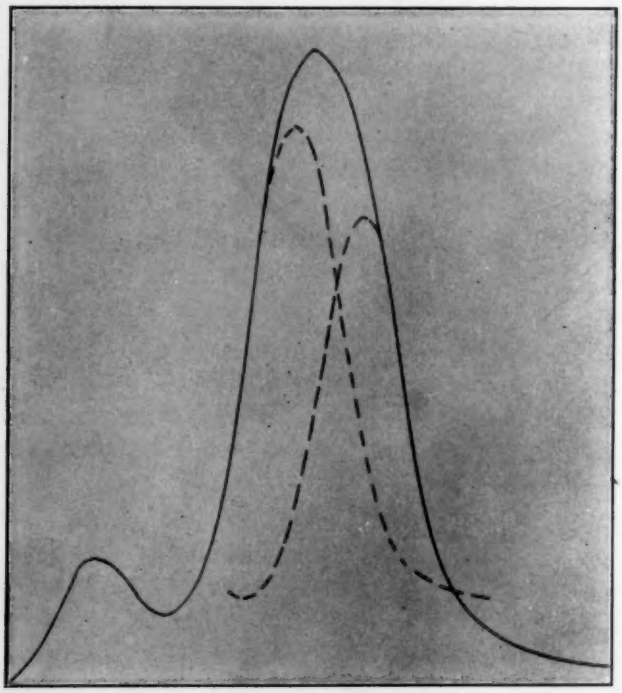


FIG. 3.—Microphotometer curve of solar pair λ4219, intensity 3 and 4, with curves of isolated lines λ4196 and λ4207, intensities 3 and 4, superimposed.

form and intensity. A simple compounding of the curves would yield too high a resultant for the center. This apparent discrepancy is referable to the fact that the areas of the registered curve, as Tugman shows, do not measure the energy of the spectral lines.

5. DEFINITIVE RESULTS

Pairs consisting of lines of intensities 3 and 4 occur more frequently than any others and offer the most favorable basis for a

comparative study. To obtain an extended series of observations, thirty pairs with components of these Rowland intensities were taken for the measurement of the separations. These are the pairs between λ 3900 and λ 5270, the separation of whose components is under 0.35 A. As the conditions upon which the discrepancies between the Rowland and the Mount Wilson values depend vary with the spectral region—the width of the lines of the same nominal intensity increasing with the wave-length—separate consideration is given to the pairs to the red of λ 4000.

In Table III are assembled the data relative to these thirty pairs, which, as far as possible, were measured on each of the six series of plates. Vacant spaces in the table mean that the resolution was not sufficient for the present purposes of measurement. The parentheses indicate that, though the measurement seemed possible at the time, the results were not considered in obtaining the means because of the lack of agreement with the more concordant results for the higher orders. Whether or not they should be included is a matter of judgment; their inclusion, however, would increase the discrepancy between the Rowland and the Mount Wilson values. The pairs are arranged in the order of decreasing separation; therefore, as one passes down the table, the first-order measures soon become uncertain, then impossible, a course followed by the other orders in succession. This furnishes the means for fixing a minimum separation measurable to the indicated degree of precision with the given resolving powers and dispersions. This is influenced, however, by the wave-length and the character and intensities of the lines. The results for the 75-foot spectrograph recorded in the twelfth column were not used in obtaining the means, as the purpose was to compare the results for the two instruments. As far as the measures are dependable they are in excellent agreement.

The magnitude of the errors in the Rowland separation depends upon the proximity of the components. For the first 6 pairs, mean separation 0.274, it is +0.003; for the following 8 pairs, mean separation 0.145, it is +0.008; and for the last 8 pairs, mean separation 0.075, it is +0.013 A, based upon the mean of the three methods used.

TABLE III

Filar-Micrometer Separation of Closely Adjacent Solar Lines

PART I. PAIRS TO THE RED OF λ 4000

Rowland		Order, n.m. and 1 mm in A for 30-Foot					Mean Δλ	Wt.	Mean Dev.	Rowland minus Mt. W.	75-Foot	Remarks
Wave-Length	Int.	I 95,000 1.80	II 190,000 0.88	III 285,000 0.56	IV 380,000 0.36	V 475,000 0.27					I 77,500 0.72	
4461.818	4	0.342	0.344	0.342	0.341	0.340	0.342	31	0.001	+0.005	0.340	
2.165	3											
4489.911	4	.339	.338	.341	.340	.340	.340	31	.002	+ .002	.339	
90.253	3											
4339.617	4	.267	.265	.264	.264	.262	.264	12	.002	+ .001	.263	
.882	3											
5079.158	3	.254	.250	.248	.249	.246	.249	17	.003	+ .002	.248	
.409	4											
4556.063	3	.238	.240	.240	.241	.238	.239	21	.002	+ .004	.241	
.306	4											
5139.427	4	.209	.212	.212	.212	.217	.212	15	.002	+ .005	.210	Poor quality
.644	4											
5107.619	4	.190	.193	.192	.194	.192	.192	27	.002	+ .012	.191	Excellent quality
.823	4											
5041.069	3	.180	.182	.185	.189	.188	.183	18	.005	+ .003	.175	Violet component double
.255	4											
4024.726	3	154	.157	.156	.154	.156	.156	21	.001	− .001	.153	
.881	4											
5169.069	3	.....	.133	.137	.137	.139	.136	19	.003	+ .015	(.124)	Difficult
.220	4											
4171.068	4	(.129)	.134	.136	.136	.136	.135	14	.002	+ .010	.132	
.213	4											

TABLE III

PART I—*Continued*

4667.626 .768	4 3	.....	(.125)	.131	.131	.133	.133	.132	16	.001	+ .010	(.125)	
4315.138 .262	3 4	.....	(0.099)	.111	.114	.119	.114	.115	19	.003	+ .009	(.097)	
4078.515 .631	4 3	.....	.....	.107	.106	.110	.112	.109	32	.003	+ .007	(.103)	
4581.575 .693	4 4	.....	.....	(0.102)	.109	.107	.106	.107	10	.003	+ .011	(.090)	
5270.438 .558	3 4	.....	.....	.....	.098	.104	.113	.104	15	.006	+ .016	.....	Very difficult
4878.313 .407	3 4	.....	.....	.....	.092	.094	.093	.093	17	.005	+ .001	.....	Very difficult
5264.329 .415	4 3	.....	.....	.....	.076	.080	.083	.079	11	.004	+ .007	.....	Very difficult
4476.185 .253	4 3	.....	.....	.....	0.069	0.069	.067	.068	2	.003	.000	.....	Very difficult
4219.516 .580	4 3	.....	.....	.....	.....	.....	.053	.053	1	.....	+ .011	.....	Very difficult
4204.101 .163	3 4	.....	.....	.....	.....	.....	.046	.046	3	.002	+ .016	.....	Very difficult
4210.494 .561	4 3	.....	.....	.....	.....	.....	0.046	0.046	2	0.002	+0.021	.....	Very difficult

TABLE III

PART II. PAIRS TO THE VIOLET OF λ 4000

Rowland		Order, n.m. and 1 mm in A for 30-Foot					Mean Δλ	Wt.	Mean Dev.	Rowland minus Mt. W.	75-Foot	Remarks
Wave-Length	Int.	I 95,000 1.80	II 190,000 0.88	III 285,000 0.56	IV 380,000 0.36	V 475,000 0.27					I 77,500 0.72	
3910.984	4	(0.158)	0.153	0.153	0.152	0.150	0.152	14	0.002	−0.001	0.154	Poor quality
1.135	3											
3956.476	4	(.116)	.121	.119	.120	.120	.120	17	.001	+ .007	.118	
.603	4											
3979.664	4	(.114)	.117	.119	.116	.118	.117	14	.002	+ .002	.119	
.783	4											
3948.818	4	.....	.099	.100	.103	.104	.102	17	.002	+ .005	(.095)	
.925	4											
3918.464	4	.....	(.091)	.098	.098	.100	.099	11	.001	.000	.091	
.563	4											
3989.912	4	.....	(.085)	.094	.094	.094	.094	14	.001	+ .005	.088	
90.011	3											
3919.208	3	.....	(0.082)	.091	.094	.094	.093	11	.002	+ .008	(0.081)	
.309	3											
3926.086	4	.....	.....	(0.069)	0.075	0.074	0.075	8	0.001	+0.004	.....	
.165	3											

As is well known, the solar lines decrease in width and the spectrograms improve in definition in passing from the red to the violet. This change is striking when one attempts measurements near the limit of resolution in the two regions in immediate sequence. In Part II of Table III are the data for 8 pairs to the violet of λ 4000, mean separation 0.107 A, for which the mean error in Rowland is +0.004 A, approximately half that for the pairs to the red of λ 4000 with the same mean separation. A difference of this order is consistent with the appearance of pairs in the two regions, for, in the violet, owing to the better resolution, the contrast between the intensities of the background on the two sides of a component is much less marked for pairs of like intensity and separation.

As the Mount Wilson measures, made under the most diverse conditions, differ systematically from the Rowland values, the assumption of corresponding errors in the Rowland tables seems a well-grounded conclusion. The establishment beyond doubt of the errors in a few cases may, however, strengthen the evidence that the discrepancies in general represent actual errors in the Rowland tables. The interpretation of the registered curves for the pairs at λ 5167 and at λ 5169 given in an earlier paragraph (p. 20) does not appear open to serious criticism, as no recognized peculiarity in the action of the registering microphotometer has the effect of displacing the axes of curves for lines so nearly resolved. An example still more free from possible criticism is supplied by the solar pair at λ 5107. These are lines of the best quality, and are completely resolved upon the Mount Wilson plates. A spectrum of the fourth order is reproduced in Plate I (*b*) and the curve in Fig. 4. The Mount Wilson micrometer measures give a separation of 0.192 A for the mean of 33 measures on the six series of spectrograms with an average deviation of 0.002 A. The separation given by the registered curve is 0.190 A, while that from the Rowland wavelengths of the constituent lines is 0.204 A, a discrepancy of +0.013 A.

Another example that appeals to the eye, even without measurement, is found in the pairs at λ 3918 and λ 3919. A spectrum of the fifth order is shown in Plate I (*c*), and the corresponding registered curves in Fig. 5. The eye decides at once which is the wider

separation, and both the micrometer measurements and the registered curves confirm it. The separations deduced from the Rowland wave-lengths are 0.099 A and 0.101 A, but the separations upon the Mount Wilson plates are 0.099 A and 0.093 A, respectively.

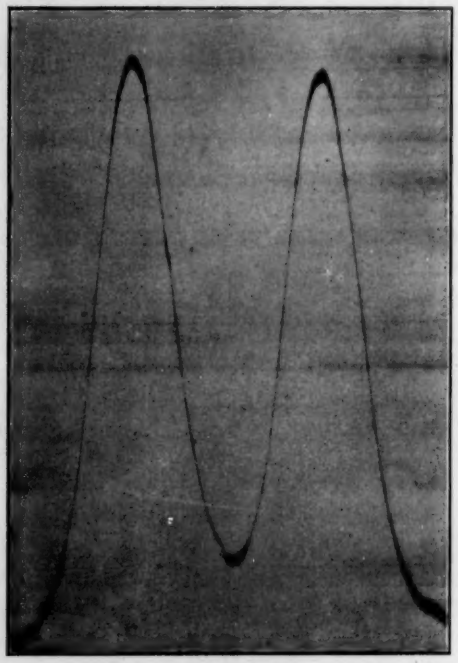


FIG. 4.—Microphotometer curve of Fe pair λ 5107.

6. THE ROWLAND PLATES

Through the kindness of Professor Ames some of the original plates taken by Rowland for the solar spectrum map were loaned to the Observatory. Two plates of the third order are especially good, showing very little grain. In the introduction to the *Preliminary Table of Solar Spectrum Wave-Lengths* Rowland says:

> In some cases when a double line is particularly difficult to separate, measurements are given on the two components and also on the line unresolved. This last measurement is placed in parenthesis between the other two measurements. Thus:
>
> | W.-L. | | |
> |---|---|---|
> | 3738.454 | 3 | d |
> | (3738.466) | 6 | |
> | 3738.505 | 2 | |
>
> means that there is a line at W.-L. 3738.466 with the intensity 6; and that with good definition this line may be resolved into two components having intensities 3 and 2 and the given wave-lengths.

Such a complex appearing in the table at λ 3926 is well resolved on the Rowland plates, more plainly resolved, in fact, than other doubles on the plates, which, from the absence of any indication to the con-

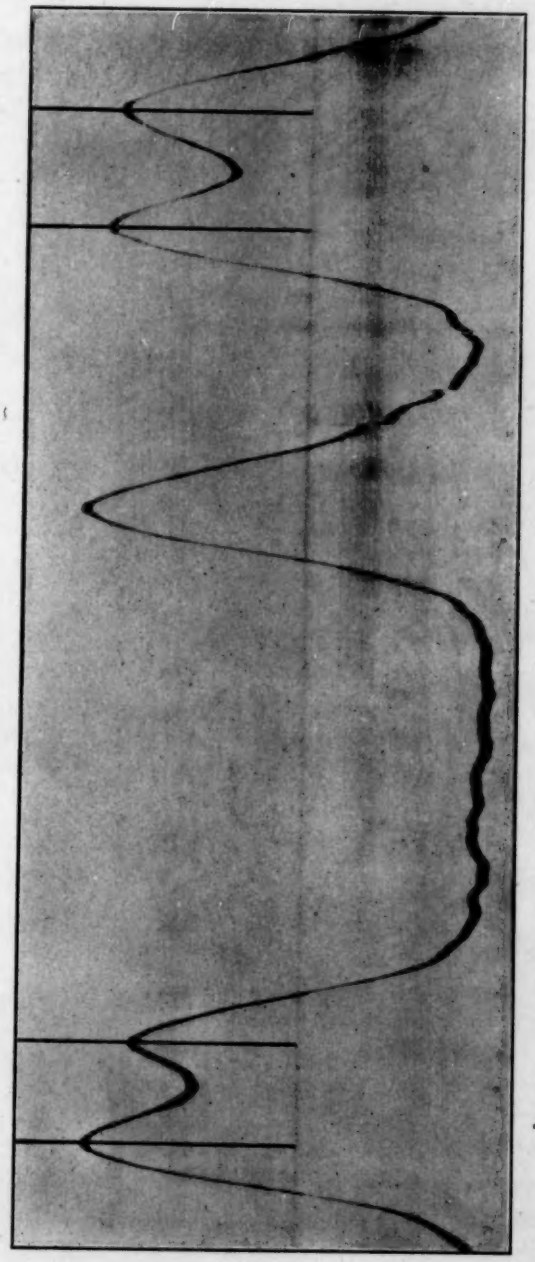


FIG. 5.—Microphotometer curve of Fe pair λ 3918 and Fe-Cr pair λ 3919

trary, were measured without difficulty. It is thought, therefore, that the quality must have been considered very satisfactory by Rowland and Jewell. Though the Rowland plates excel the Mount Wilson plates (Seed's Process) in fineness of grain, the slightly

TABLE IV

COMPARISON BETWEEN MOUNT WILSON AND ROWLAND THIRD-ORDER PLATES

Wave-Length	Int.	Resolution		Mt. Wilson Measures		Rowland Table Δλ	Rowland Table *minus* Mt. W. Plate
		Rowland 1 mm = 0.64 A	Mt. Wilson 1 mm = 0.57 A	Rowland Plate Δλ	Mt. Wilson Plate Δλ		
3910.984	4	$s=c$	$s=c$	0.151	0.153	0.151	−0.002
.135	3						
3914.426	3	$s<<<<<c$	$s<<<<<c$	.046	.044	.051	+ .007
.477	2						
3918.464	4	$s<<c$	$s<c$	.101	.098	.099	+ .001
.563	4						
3919.208	3	$s<<<c$	$s<<c$	.094	.091	.101	+ .010
.309	3						
3926.086	4	$s<<<<c$	$s<<<c$	.075	.069	.079	+ .010
(.123)							
.165	3						
3948.818	4	$s<<c$	$s<c$	.106	.100	.107	+ .007
.925	4						
3956.476	4	$s=c$	$s=c$	.124	.119	.127	+ .008
.603	4						
3979.664	4	$s=c$	$s=c$	.120	.119	.119	.000
.783	3						
3989.912	4	$s<<<c$	$s<<c$	.094	.094	.099	+ .005
.011	3						
4024.726	3	$s=c$	$s=c$	0.154	0.156	0.155	−0.001
.881	4						
Mean ...							+0.0045

larger scale of the latter compensates for this, so that, in fact, they are somewhat easier to measure. For comparison ten pairs were measured upon both the Rowland and Mount Wilson plates. The data are shown in Table IV. In the third and fourth columns is given an indication of the resolution. For four pairs $s=c$; the

intensity in the space between the components equals that of the continuous spectrum background at their outer edges, i.e., the resolution is complete. The pair at λ 3914 shows only the merest trace of resolution. For the other five pairs the resolution upon the Mount Wilson plates, though far from complete, is a degree better than upon the Rowland plates.

(*a*) The Rowland values exceed the Mount Wilson measures on the Rowland plates by +0.0023 A; this suggests that the personal equation of the observer plays an important rôle. (*b*) The Mount Wilson measures on the Rowland plates exceed the measures on the Mount Wilson plates by +0.0022 A, i.e., owing to the better resolution the influence of an adjacent line is less on the Mount Wilson plates with a correspondingly lessened tendency toward overspacing. The two effects (*a*) and (*b*) account for the Rowland *minus* Mount Wilson mean of +0.0045 A. The discrepancies between the Mount Wilson and the Rowland separations measured upon the same plates are most manifest for pairs near the limit of resolution, thus:

Fairly Resolved R.−Mt. W.		Near Limit of Resolution R.−Mt. W.	
λ 3910	0.000	λ 3914	+0.005
3918	−0.002	3919	+0.007
3948	+0.001	3926	+0.004
3956	+0.003	3989	+0.005
3979	−0.001		
4024	+0.001		

7. DISCUSSION

In view of the diversity in the methods of measurement employed at Mount Wilson and the wide range in the scale and resolution of the spectrographic material, it seems probable that the effects due to personal equation have been greatly reduced, if not eliminated, and that the differences between the Rowland and Mount Wilson measures represent systematic errors in the former. A tendency toward over-separation of the components of close pairs appears to accompany departure from the conditions for finest definition. As the Rowland plates were taken for the average run of lines, it seems probable that the errors for closely

adjacent lines would be systematically positive. That the evidence from the Mount Wilson observations is cumulative is seen from the data in Table V, where the results from various lines of investigation are assembled. The magnitude of the error depends upon several factors, the proximity, the intensity, the character, and the wave-length of the components. For lines of like intensity and character in a definite spectral region the error increases with decrease of separation; for lines of the same separation, character, and region it increases with the intensity; for lines of the same separation, character, and intensity it increases with the wave-length.

TABLE V

RÉSUMÉ OF DATA

Source	Table I	Table II			Table III		Table IV		Pairs 5167–5169
Wave-length	4465	4685	4680	4570	4640	3940	3960	3940	5168
Intensity	2.5	3.5	3.5	3.6	3.5	3.8	3.8	3.1	6.5
Separation	0.087	0.073	0.274	0.145	0.075	0.107	0.125	0.080	0.149
R.–Mt. W.	+0.006	0.012	+0.003	+0.008	+0.013	+0.004	0.000	+0.005	+0.017

The difficulties of measurement and the systematic character of the errors encountered in this investigation occur in the determination of the absolute wave-lengths of lines with closely adjacent companions. Differential measures upon similar spectra such as those of the two limbs of the sun would not be subject to such systematic errors, provided the spectra were of equal intensity, a condition that gains in importance when the measurements are upon lines not completely isolated.

A résumé of the circumstances that accompany the systematic differences between the Rowland and the Mount Wilson measurements suggests as a cause a personal equation whose influence appears in estimating or neglecting the effect of contrast. Early in this investigation a tendency toward an increase in the measured separations accompanying a lessened intensity of the photographic plate became evident. For example, six Mount Wilson observers, including the four with the widest experience in solar work, measured the separation of the pair at λ 5455 upon two spectra identical

except for a slight difference in intensity. The measurements agree in showing a separation upon the weaker exposure greater by 0.013 A; an appeal to the microphotometer curves supports the smaller value.

An indication of wider import is that a falling off in definition due to any cause is a condition productive of the errors in question. It is immaterial whether the decrease in the sharpness of the lines with the consequent loss in definition is caused by a slight change in focus or a widening of the slit. Two third-order spectrograms, No. 56 and No. 199, throw light on the question. Upon Plate No. 56, considered at the time to be of excellent quality, seven close pairs were measured by three experienced observers with fairly concordant results. Later, spectrogram No. 199 was taken in the same region with a two-normal slit and a backed plate. The plates were taken with the same focal setting and are practically of the same intensity, but No. 199 is very superior in definition. The five exposures were measured by two of the previous observers with very concordant results, which appear in Table VI, together with those from a fifth-order spectrogram. The separations given by No. 56 are consistently larger than those from No. 199, the average difference being +0.008 A. The differences, moreover, are greater for the pairs nearest to the limit of resolution. For four such pairs it is +0.012, while for the three of better quality it is +0.004 A. The appearance of pairs very near the limit of resolution is extremely sensitive to instrumental conditions, 1 mm change of focus in the 30-foot spectrograph—one part in nine thousand—being recognizable in the spectrum of close pairs in which both lines are of good quality. The close agreement between Plate No. 199 and the fifth-order plate furnishes a criterion for deciding between the two third-order plates and for the rejection of the measurements made upon No. 56.

An explanation of these discrepancies based upon the psychological effects of contrast appears to be in harmony with the observations. On line 5167, Plate I, represented by the curve in Fig. 1, the intensity at the symmetrical points *B* and *C* is the same, but *B* is seen against the relatively strong continuous background at *A*, while *C* is seen against the weaker background at *D*.

Under the influence of the difference in contrast, *B* appears of greater intensity than *C* and is of greater influence in estimating the position of the maximum, which is consequently moved outward. A similar effect operates in fixing the maximum of the adjacent line; the combined result is therefore a separation more or less in error in the positive direction, the amount depending upon the personal equation of the observer. The observations show that the increase in this contrast difference, which occurs whenever the intensity

TABLE VI

INCREASE IN SEPARATION ACCOMPANYING DECREASE IN DEFINITION

λ	Int.	Plate 56	Plate 199	Plate 56 − Plate 109	Order V
4455.980 .064	2 3	0.086	0.074	+0.012	0.073
4457.600 .712	2 2	.112	1.08	+ .004	.106
4459.199 .301	2 3	.104	.091	+ .013	.095
4464.844 .938	2 1	.093	.091	+ .002	.088
4472.884 .967	1 0	.082	.076	+ .006	.077
4476.185 .253	4 3	.082	.070	+ .012	.066
4482.338 .438	5 3	0.104	0.094	+0.010	0.092
Mean.....	.....	0.095	0.086	+0.008	0.085

in the region of overlap is decreased relatively to that of the continuous spectrum, is accompanied by increase in the measured separation. The influence in question appears in the case of the pairs at λ 5167 and at λ 5169, where the micrometer measures always exceed those made by the registering microphotometer (p. 20); also in Table II, where the micrometer results average larger by 0.005 A than those found by analysis, and in Table VI, where the spectrogram taken with a wider slit gives separations greater by 0.008 A than Plate No. 199.

When the spectrograms differ only in intensity, the matter is complicated by photographic effect. Whether or not increase of exposure lessens the difference in contrast on the two sides of a line in a close pair depends upon where on the characteristic curve of the plate the density of the continuous spectrum and that of the region of overlap, respectively, fall—a question not yet investigated. The effect studied by Eberhard[1] may also be of influence, though from an investigation by Koch[2] it appears to be negligible.

The wave-lengths of 54 solar lines separated 0.25 to 0.50 A from adjacent lines have been measured upon plates of high dispersion, the neighboring free-standing lines being used as standards. The mean variation from Rowland is ±0.003 A. As over 250 lines were used for reference, it appears that the accidental errors in the relative wave-lengths of the *Preliminary Table of Solar Wave-Lengths* for lines practically isolated are much less than ±0.01 A, a value frequently assumed. It is probable, as Frost and Adams[3] remark, that errors of this magnitude occur but rarely, and mainly then for lines whose measurement is intensely difficult, such as the very weak or strong lines. The proximity and character of neighboring lines introduced disturbing elements that have apparently affected the results for closely adjacent lines and given the errors a systematic character. Information upon which to base a discriminating judgment has not been available, and the user of the table, who has not solar spectrograms of high dispersion at command, could hardly do otherwise than ascribe equal precision to the data for lines that appear in the table without distinguishing signs, though the errors for certain classes of lines are relatively large and systematic.

SUMMARY

1. The measurement of solar lines near the limit of spectrographic resolution is a matter of extreme difficulty and liable to be systematically in error.

2. The Mount Wilson separations have been determined, as far as possible, upon each of five series of spectrograms with dispersions varying from 1 mm = 1.8 A to 1 mm = 0.23 A.

[1] *Phys. Zeit.*, 13, 288, 1912. [2] *Annalen der Physik*, 42, 1, 1913.

[3] *Publications of the Yerkes Observatory*, 2, 155.

3. Separations obtained from curves produced by the registering microphotometer are, for incompletely separated components, smaller than those found by filar-micrometer settings.

4. For pairs near the limit of resolution analysis based upon microphotometer curves, settings upon the edges of the doublet and of lines similar to its components, and ordinary filar-micrometer measurements have been employed. The first appears most reliable and yields smaller values than the third.

5. Separations equal to the theoretical spectrographic resolution, though serving to detect duplicity, are not sufficient for filar-micrometer measurements of wave-lengths to the third decimal place in angstrom units.

6. Spectrograms of the finest definition yield the lowest values for the separation of the components of doublets near the limit of resolution.

7. Filar-micrometer measurements of the separation between the components of close doublets varies with the width of the slit, the precision of the focal settings, and the density of the spectrograms.

8. Whatever decreases the intensity of the common region relatively to that of the continuous spectrum produces a tendency on the part of the measurer toward increased separation. This seems to be an effect of contrast, the observer locating the maximum nearer that edge of the line for which the contrast is greatest, i.e., nearer the free edge.

9. The differences between the Rowland and Mount Wilson determinations of the separation of the components of close pairs of solar lines are systematic. For pairs to the red of λ 4000, component intensities 3 and 4, with mean separations of 0.276 A, 0.145 A, and 0.075 A, the Rowland values exceed the Mount Wilson values by +0.003 A, +0.008 A, and +0.013 A, respectively.

10. That these differences are errors in the Rowland values is made probable by the agreement between the diverse methods used, by the concordance between the spectrograms of different orders, and by the tendency toward over-separation with any departure from best conditions.

MOUNT WILSON SOLAR OBSERVATORY
June 1916

ON THE TEMPERATURE AND RADIATION OF THE SUN

By C. G. ABBOT, F. E. FOWLE, and L. B. ALDRICH[1]

The paper of Felix Biscoe[2] on this subject seems to require comment from the point of view of the Smithsonian Astrophysical Observatory.

We are not greatly concerned with the part relating to the temperature of the sun, though that is based on our observations. We of course dissent altogether from the treatment of them. Even if one should admit that the properties of the photosphere and solar atmosphere are as assumed by Biscoe,[3] the conclusion stated by him[4] does not seem to us to follow. Like many others, he treats the transmissions of the solar atmosphere as if all that is necessary is to deal with a beam of parallel rays coming directly through the solar atmosphere. He neglects altogether the rays *scattered into the beam*. It is well known that so slight an envelope as our terrestrial atmosphere scatters something like 10 per cent out of the direct solar beam. If we compare the gigantic sun with the pigmy earth it is apparent that even a "thin" atmosphere on the sun must scatter much more. Under solar conditions, contrasted with terrestrial ones, scattering takes place on rays arising from every direction instead of from one direction alone. Accordingly the beam which seems to come from the center of the sun's disk, as (to use a homely illustration)the handle of an umbrella comes from its center, really comes largely by scattering from all sides, just as the strength of the umbrella handle arises from its ribs. On this account we regard Biscoe's "coefficients of transmission of the solar atmosphere" as quite without foundation.

As for his basic assumption of a sharply defined "black-body" photosphere radiating through a thin gaseous envelope which takes no sensible part in the radiation, we have long regarded this as

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, 43, 197–217, 1916.

[3] *Ibid.*, p. 205, lines 5–11; p. 199, lines 21–22.

[4] *Ibid.*, p. 199, lines 23–24.

improbable, and it grows more improbable with every new discovery. Our views on this subject are so fully published that we need not occupy space here for them.

Turning now to Biscoe's observations, nothing in our experience has prepared us for such a wide range of results as his values of "S_0," the apparent solar constant from pyrheliometry alone, published on p. 211 of his article. If we may express our opinion, either the observations are erroneous or they are made on many days that could be seen by the eye to be unsuitable. He very justly concludes that they do not warrant the belief that their variations are exclusively of solar origin.

Biscoe then refers to our solar-constant measurements made in 1911 and 1912 at Mount Wilson and Bassour, and says that he attributes their variations, which we believe to indicate a variability of the sun, to terrestrial influences and erroneous methods of reduction. He believes that he has shown that when we get high values of the solar constant it is because the atmospheric transmission is low, and vice versa. This conclusion he regards as strengthened by a reference to several Washington observations.

Mr. H. Knox Shaw in a paper of high merit[1] discusses a similar suggestion, and points out in particular the apparently untrustworthy character of our results for the autumn of 1911, both at Mount Wilson and at Bassour.

We admit at the start:

1. For purposes of determining the variability of the sun, Washington observations are too inaccurate to be worth consideration.

2. In the autumn of 1911 the sky, both at Mount Wilson and at Bassour, was so often overcast by cirrus clouds that a large portion of the days had to be discarded, and others of them are really unfavorable, though they were not obviously so to the eye.

3. In the year 1912 the volcano of Katmai made the atmosphere so turbid that the purpose of our expeditions was near to being defeated. We observed the sun's variability under a most unusual and unfortunate handicap.

4. Our own investigations have convinced us that some of the days we have included in Mount Wilson results of each and every

[1] *Helwan Observatory Bulletin*, No. 17.

year are affected by unfavorable atmospheric conditions. Such days are likely closely to precede and follow storms, periods of cloudiness, or periods of great humidity. We have not thought best hitherto utterly to exclude such days, partly because it is difficult to frame a suitable criterion for exclusion, and partly because we alone are making solar-constant observations, and an unsatisfactory value may be better than none.

These admissions made, we stand by the variability of the sun; by the proof of it in the work of 1912; by the close accuracy of our mean value of the solar constant of radiation; and by the soundness of our methods of solar-constant determination.

The variability of the sun is now confirmed[1] by (*a*) Mount Wilson observations of the solar constant, (*b*) comparison of Mount Wilson and Bassour observations, (*c*) comparison of Mount Wilson and Arequipa observations, (*d*) comparison of Mount Wilson and magnetic observations, (*e*) comparison of Mount Wilson solar-constant work with Mount Wilson solar-contrast work. The cumulative effect of this evidence is overwhelming.

We do not give much weight to the 1911 comparison of Bassour and Mount Wilson work. As for 1912, as shown by Shaw, the comparison yields a correlation coefficient of 58 per cent with a probable error of 7.9 per cent. Biscoe will have us admit that this result depends on the combination of three correlations: (*a*) correlation of solar-constant and atmospheric transmission at Mount Wilson; (*b*) correlation of solar-constant and atmospheric transmission at Bassour; (*c*) correlation of atmospheric transmission at Mount Wilson with atmospheric transmission at Bassour. We call attention to Figs. 1 and 2, which show the first two of these supposed correlations graphically. The factors of correlation are respectively:

$$(a) \quad r = +0.05 \pm 0.12$$
$$(b) \quad r = -0.26 \pm 0.11$$
$$(c) \quad r = +0.59 \pm 0.08$$

Correlation (*a*) would result $r = -0.27 \pm 0.12$ if one day should be omitted. The large correlation coefficient (*c*) is of course due to

[1] See *Annals of the Smithsonian Astrophysical Observatory*, **3**; *Smithsonian Miscellaneous Collections*, **65**, Nos. 4 and 9; and **66**, No. 5; *Terrestrial Magnetism and Atmospheric Electricity*, **20**, 143, 1915.

the volcano, and shows how seriously the undertaking was handicapped. But when all is said against the work, we think no one can fairly draw the conclusion that the combination of the two low and inaccurate correlations (*a*) and (*b*) with the very high correlation (*c*) can reasonably be expected to account for the very high correlation of 0.58±0.08 between the solar-constant values themselves as computed by Shaw. It is further to be noted that not only are higher values found at Bassour when higher ones are

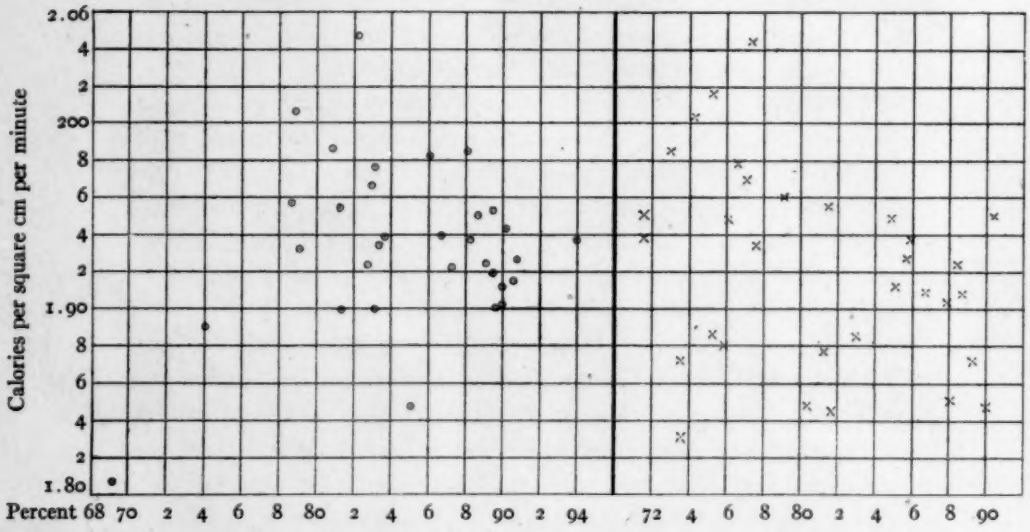


FIG. 1.—Mount Wilson values of solar constants for 1912 and coefficients of apparent atmospheric transmission.

FIG. 2.—Bassour values for 1912

found at Mount Wilson, but that the best representative straight line of Fig. 15 (*Annals of the Smithsonian Astrophysical Observatory*, 3) is at 45° inclination, as predicted according to the hypothesis of solar variation.

Critics should recall besides that if the sun really varies, higher real values of solar radiation may have occurred during part of the time when the atmospheric turbidity due to the eruption of Katmai was great. In that case the small negative correlations (*b*) and (*a*), as altered by removing one day, would be largely apparent, not real. Hence the results of 1912 cannot be regarded as proving

that higher solar-constant results functionally attend lower atmospheric transparency.

In order to see if there is a real negative correlation of our Mount Wilson solar-constant values with the atmospheric transmission, we have selected from all the years 1910–1914 a considerable number of days to which no exception can be taken, either as to atmospheric or as to instrumental conditions. For this purpose we have excluded days which were on the verge of approaching or receding cloudiness or great humidity, days in which the observed solar constant was greatly different from values immediately before and after, days in which cirrus clouds appeared, and days when there were any observations of medium or low grade. The days selected are as shown in Table I. From these data we find the following

TABLE I

MOUNT WILSON RESULTS, UNEXCEPTIONABLE CONDITIONS

Date	Solar Constant	Apparent Transmission	Date	Solar Constant	Apparent Transmission
1910 Oct. 6...	1.920	0.904	1913 Sept. 28..	1.855	0.889
7...	1.894	.915	29..	1.882	.885
8...	1.892	.906	Oct. 19..	1.873	.889
9...	1.921	.893	20..	1.868	.906
10...	1.940	.906	21..	1.912	.887
Nov. 7...	1.905	.929	22..	1.893	.885
8...	1.907	.929	23..	1.871	.897
			24..	1.882	.900
1911 July 2...	1.911	.885	25..	1.850	.900
3...	1.911	.887	26..	1.871	.887
Aug. 10...	1.927	.933	Nov. 8..	1.902	.906
12...	1.949	.914			
14...	1.925	.912	1914 June 13..	1.943	.895
			14..	1.944	.895
1912 June 7*..	1.939	.908	23..	1.943	.904
18...	1.928	.906	24..	1.936	.895
			Sept. 2..	1.948	.881
1913 Aug. 3...	1.928	.861	3..	1.942	.900
4...	1.916	.867	14..	1.954	.885
6...	1.913	.875	15..	1.965	.887
12...	1.940	.885	20..	1.936	.916
13...	1.927	.890	21..	1.960	.906
14...	1.955	0.883	Oct. 12..	1.951	0.904

* See *Annals of the Smithsonian Astrophysical Observatory*, 3, p. 121, note.

negligibly small correlation coefficient between solar-constant values and apparent atmospheric transmission coefficients:

$$r = -0.06 \pm 0.10.$$

Hence we may conclude that while some of the very best Mount Wilson solar-constant results indicate large solar variation, they show no dependence on the atmospheric transparency.

In conclusion we take this opportunity of expressing our sense of the high value which would attach to the establishment of several additional spectro-bolometric stations for solar-constant observing. Fortunately the Smithsonian Institution will establish such a station in South America soon. But the variability of the sun is so considerable, and its consequences may well be so interesting, that we should welcome steps leading to accurate measurements of the solar variability in at least two other widely separated and highly favorable regions. As we have admitted above, not all cloudless days give good values, even on Mount Wilson. Four independent accurate determinations of the solar constant would not be superfluously many from which to fix the daily values of the intensity of solar radiation outside our atmosphere.

SMITHSONIAN ASTROPHYSICAL OBSERVATORY
WASHINGTON, D.C.
May 1916

INTENSITY OF THE CONTINUOUS SPECTRUM OF STARS AND ITS RELATION TO ABSOLUTE MAGNITUDE[1]

By GEORGE S. MONK

Some time ago a comparison[2] was made by Mr. Adams of the intensity of the continuous spectrum of pairs of stars of large and small proper motion photographed upon the same plate, which showed that stars of small proper motion are relatively weaker in the more refrangible portion of the spectrum. Later, a large number of spectrograms obtained for radial velocity were compared and the results tabulated.[3] This comparison showed (1) that in types Fo to K4 stars of small proper motion have spectra which are weaker in the violet than stars of large proper motion; (2) that this difference increases with advancing type from F to K.

Accepting proper motion as an indication of distance, the first result might be interpreted as a consequence of the scattering of light in space; but the second result points to the fact that the absorption in the violet portion of the spectrum is due, at least in part, to physical conditions in the stars rather than to scattering.

In continuation of this method of comparison, the density of the continuous spectrum of most of the stars on the radial-velocity program has been estimated. The method used has been fully described in the second of the articles previously referred to, the only difference being that a negative with five successive exposures of Arcturus has been used as a standard for comparison instead of the plate of α Tauri previously used. Briefly, the method consists of comparing the photographic densities of stellar spectra with the standard plate of Arcturus. The standard plate is then measured under a microphotometer and the densities obtained at the points of comparison. From the estimated relation between

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 119.

[2] *Mt. Wilson Contr.*, No. 78; *Astrophysical Journal*, 39, 89, 1914.

[3] *Mt. Wilson Contr.*, No. 89; *Astrophysical Journal*, 40, 85, 1914.

the densities of the standard plate and that of any other star, the actual densities of the spectrum of the star may be deduced. In order to check the accuracy of this method, twenty spectra selected at random were measured under a microphotometer; the results were found to be in good agreement with those obtained by comparison with the standard plate of Arcturus. The densities on about 1200 plates have been estimated and reduced by this method.

It was necessary to reject many plates for the following reasons: (*a*) presence of clouds or haze during exposure; (*b*) great zenith distances; the average zenith distances for groups of stars compared have been made to agree closely; (*c*) photographic blemishes on negatives; (*d*) extremely strong or weak negatives; the average intensities of any two groups compared were made to agree.

By the use of these restrictions, it is believed that the disturbing effects which must be present in a mass of material obtained under such widely differing conditions have been eliminated to a considerable extent. The plate numbers which were used in tabulating the measures show that no one group of stars was photographed within a short range of time, so that variations in color-sensitiveness of the emulsions should not affect the results materially. In most cases two or three plates of the same star have been measured and the mean values of the densities used.

These measures were tabulated for each type, the stars being grouped by large and small proper motion. The results appear in Table I. Slight corrections have been applied to the densities to make the values at λ 4930 equal for each spectrum type, thus affording a direct comparison at λ 4250.

These results are in the general direction of those of Adams and Kohlschütter, but show the connection of proper motion with absorption in the violet and the increase of this effect with spectrum type to a less extent. Soon after Table I had been prepared, Mr. Adams completed his determination of the absolute magnitudes of certain F5–K9 stars on the basis of differences in spectrum lines.

A comparison of the results of Table I with these new data gave Table II, in which "absorption in violet" is the amount of density by which the part of the spectrum about λ 4250 is weaker than that about λ 4930.

This is suggestive of a relationship between absorption in the violet and absolute magnitude—a result which substantiates that obtained by Adams and Kohlschütter, who compared stars of average apparent magnitude 6.0 having small proper motion with

TABLE I

Average Type	Average μ	Density at λ 4250	Density at λ 4930
F1.....	0".020	0.41	0.35
F3.....	0.330	0.39	0.35
F6.....	0.015	0.40	0.36
F7.....	0.370	0.42	0.36
G3.....	0.018	0.27	0.38
G2.....	0.600	0.33	0.38
G7.....	0.012	0.29	0.43
G7.....	0.660	0.33	0.43
K1.....	0.017	0.23	0.39
K1.....	0.600	0.28	0.39

stars of fainter average apparent magnitude and large proper motion.[1] Consequently their differences in absolute magnitude were greater than those for the stars of Tables I and II, which were based on proper motion alone, apparent magnitude being disregarded.

TABLE II

Average Type	Absorption in Violet Large μ − Small μ	Differences of Absolute Magnitude Large μ − Small μ
F6–7.....	+0.02	+1.8
G3–2.....	0.06	4.0
G7.....	0.04	2.7
K1.....	+0.05	+4.4

All of the density measures which could be so used were then tabulated according to absolute magnitudes, giving, as a final result, Table III. The average deviation of a single star from the mean "absorption in violet" in the last column of the table is ±0.09.

[1] *Mt. Wilson Contr.*, No. 89, p. 1; *Astrophysical Journal*, 40, 385, 1914.

Few stars were available having A-type spectra and large proper motion. A comparison of this scanty material shows very little

TABLE III

Type	No. of Stars	Average Type	Average μ	Average M	Absorption in Violet
F3–F9.....	18	F6.1	0".010	+1.7	+0.01
	33	F6.0	0.500	+4.8	−0.05
G0–G4.....	17	G2.7	0.015	+0.6	+0.12
	28	G2.0	0.660	+5.2	+0.02
G5–G9.....	51	G7.0	0.011	+0.9	+0.20
	22	G7.2	0.690	+6.0	+0.07
K0–K4.....	26	K1.4	0.017	+1.5	+0.32
	18	K2.5	0.680	+6.6	+0.16
K5–K9.....	.....	.....	.....	.....	.....
	10	K7.0	0.700	+7.7	+0.22

difference in absorption, the spectra of stars which were used having large proper motion being slightly weaker in the violet.

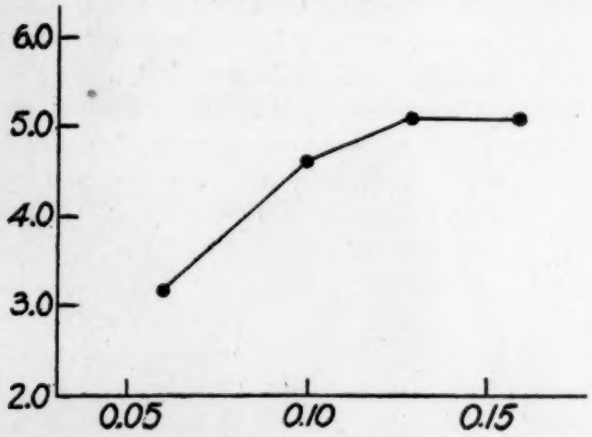


FIG. 1.—Abscissae: Differences of photographic density in the violet for stars of large and small absolute magnitude.
Ordinates: Differences in absolute magnitude.

This result, together with those expressed by Table III, is similar to that obtained by Van Rhijn.[1]

[1] Derivation of the Change of Color with Distance and Apparent Magnitude (dissertation, Groningen, 1915), p. 73.

Plotting the differences in M for each type against the differences in absorption in the violet, the curve in Fig. 1 is obtained. The relationship which seems to exist between absolute magnitude and the relative weakness in the violet of the stars having small

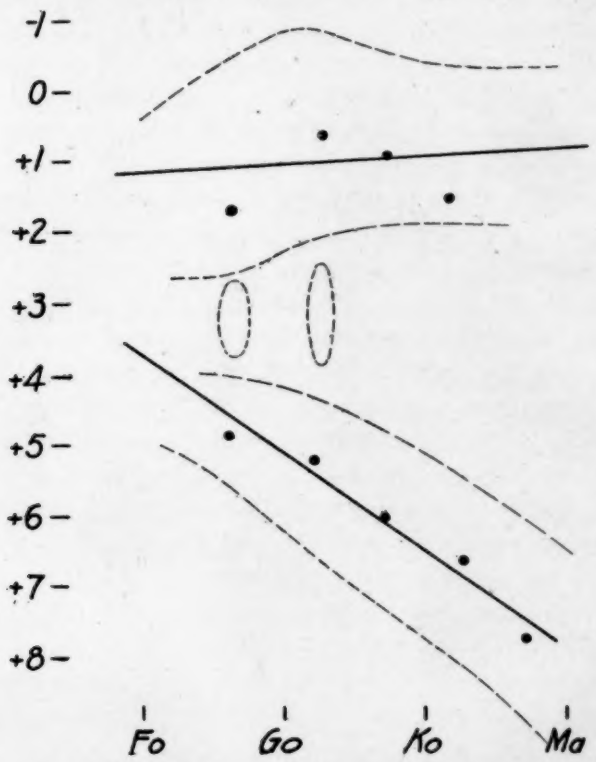


FIG. 2.—Variation of absolute magnitude with spectrum type

proper motion leads to the conclusion that at least the greater part of this effect is due to differences in absolute magnitude rather than to absorption of light in space.

No stars of large absolute magnitude between K_5 and K_9 were originally available. Seventy-six additional stars of small average proper motion and about 1.5 magnitudes fainter in apparent magnitude than the stars having small proper motion used above have been measured. Mr. Adams has found their average abso-

lute magnitude to be +2.1, about one magnitude fainter than those used in Table III. Although the values for the violet absorption fall between the two groups compared, these stars have been omitted, in part because of uncertainty in their proper motions, and in part because many of the photographs are of inferior quality.

An interesting relationship is shown in Fig. 2, in which the average absolute magnitudes of the stars used are plotted against spectrum type. The faint dotted lines show the range of magnitude within each group. The dotted ovals represent seven F and four G stars measured, but not used in this discussion. It will be seen that there is an absence of intermediate magnitudes for the latest types, a result in agreement with the hypothesis of giant and dwarf stars as discussed by Hertzsprung and Russell. It is probable that the greater irregularity of the curve in the case of the stars of small proper motion is due mainly to the relative uncertainty in the determination of their absolute magnitudes.

The photographs of spectra used in this discussion were obtained under a wide variety of conditions, and the results, accordingly, are to be considered mainly from a qualitative point of view. It seems reasonable to conclude from them, however, that with the aid of photographs taken with this purpose directly in view, the relative intensity of the violet portion of the spectrum, together with spectrum type, might be employed to provide values of absolute magnitude of a fair degree of accuracy.

I am indebted to Mr. Adams for much valuable criticism during the progress of the work and in the preparation of this paper.

MOUNT WILSON SOLAR OBSERVATORY
May 1916

THE PERIOD OF U CEPHEI

By MARTHA BETZ SHAPLEY

Throughout an interval of seventeen years Wendell at Harvard, using a polarizing photometer, made systematic measures of the eclipsing variable U Cephei for the special purpose of investigating variations in its light-period.[1] His observations constitute one of the most accurate means now available for the study of perturbations in double-star systems, for the series is not only of long duration but also peculiarly homogeneous—instrument, method of observing, and comparison star remaining unchanged. Moreover, because of the high accuracy of the measures, others made during the interval covered by them need not be considered, and earlier and later observations need be used only to check and extend the results obtained from Wendell's work. The present discussion of this extensive series was undertaken upon the suggestion of Professor E. C. Pickering.

Nearly all the available observations of U Cephei from the date of its discovery in 1880 were discussed thoroughly by Chandler[2] in 1902. He found no definite evidence of variability in the length of the period. The later Harvard observations, however, failed to conform with Chandler's period and in 1909 Wendell published a new linear formula for the light-variations.[3] His observations, begun in 1895, were continued until 1912, but no further discussion of the period was based upon them and no attempt has been made to harmonize the earlier and the later work.

As the light of U Cephei is constant at minimum for about two hours, it is not expedient to attempt a direct determination of the actual time of zero phase. Wendell adopted the plan of observing mainly the steepest part of the ascending or descending branch.

[1] *Harvard Annals*, **69**, Part I, p. 58, 1909, and Part II, p. 135, 1914.

[2] *Astronomical Journal*, **23**, 227, 1903. Yendell, Wilsing, and others had previously investigated the light-elements.

[3] *Harvard Annals*, **69**, Part I, p. 96, 1909. This formula is the one now used in the ephemeris in the *Vierteljahrsschrift*.

Usually four or five sets of measures were made while the variable was between the eighth and ninth magnitudes; and at nearly every epoch at which the star was observed the time of a given magnitude, say $8^{M}.40$, can be determined from his observations with an

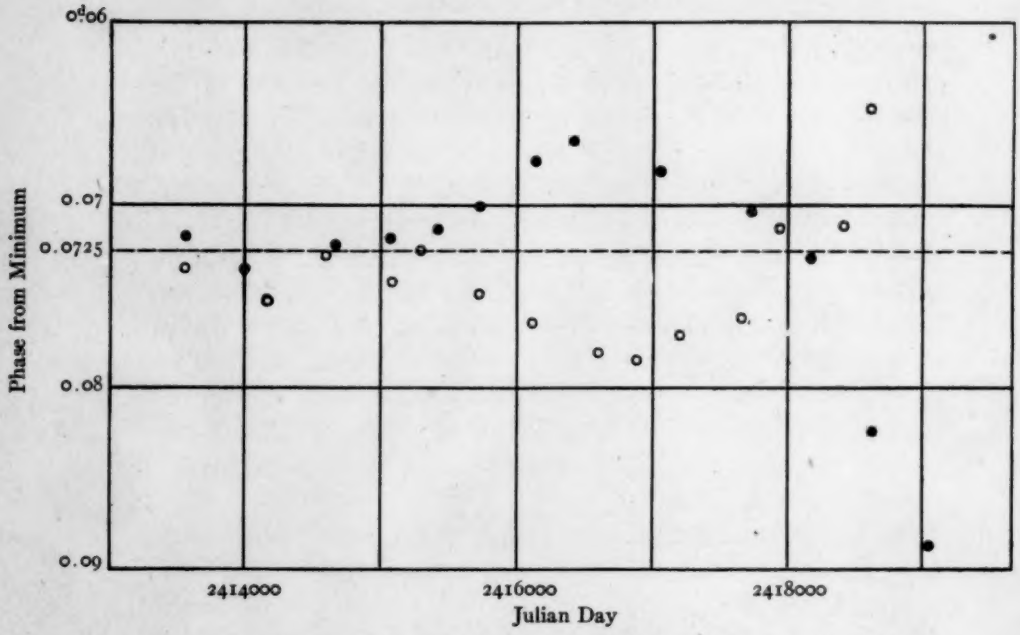


FIG. 1.—Deviations of Wendell's observations of U Cephei from his linear elements
● = Ascending branch
○ = Descending branch

uncertainty of less than a minute. The published phases of all of Wendell's measures were computed from the formula

$$\text{Min.} = \text{J.D. } 2407890.3007 + 2^{d}.4928840 \cdot E,$$

with zero phase at the midpoint of minimum light.

The determination of the mean time of crossing $8^{M}.40$ involved the plotting of each night's work and the superposition and appropriate adjustment of a mean curve upon the individual observations in the chosen interval ($7^{M}.9$ to $8^{M}.9$). All the comparisons made by Wendell, 17,296 in number, are collected into 1081 sets. Of these, 450 fall within the adopted magnitude limits and define the mean

times at 103 different epochs. Table I contains, in order of Julian Day, the data relative to these determinations. The letter in the third column indicates whether the ascending or descending branch was observed at the corresponding epoch.

The 103 determinations in Table I, weighted according to the number of observations involved, were grouped into the 27 normal points of Table II. Ascending and descending branches, which were not combined in forming means, are shown in Fig. 1 as dots and circles, respectively. Variations in the mean phases of both steep branches are immediately apparent from the indicated curves. A study of their form and similarity shows that the variation is not in the width of the minimum, nor due to variability of the comparison star, but is rather a definite change in the length of the light-period—a change, moreover, which is obviously not merely an apparent one due to an erroneous linear formula.

From a discussion of the residuals the average probable errors of an observed phase are found to be as follows:

From one observation, ascending branch	$\pm 0^d.00102$
descending branch	± 0.00097
From one epoch of minimum, ascending branch	± 0.00048
descending branch	± 0.00047
From one normal, ascending branch	± 0.00023
descending branch	± 0.00025

Thus the probable error of an average point plotted in Fig. 1 is about 21 seconds of time. But the diagram shows that the greatest deviation from Wendell's elements is about 24 minutes (taking the true time of crossing $8^M.40$ as $0^d.0725$, indicated by the dotted line). Throughout the seventeen-year interval the two branches of the curve are everywhere in almost perfect agreement as to the amount and sign of the deviation from a uniform period; and the amplitude of the variation from any possible uniform period is as much as 30 times the average probable error of a single normal point.

An investigation of magnitudes at maximum and minimum light yielded some evidence of change in the range of U Cephei at principal minimum, but the data in Table III show that it has no

TABLE I

No.	Julian Day	Branch	No. Obs.	Mean Phase of Crossing 8M40	No.	Julian Day	Branch	No. Obs.	Mean Phase of Crossing 8M40
1...	2413531	D	4	0d0720	53..	2416059	A	4	0d0680
2...	"	A	3	.0705	54..	6136	D	4	.0794
3...	3581	D	3	.0752	55..	6151	"	4	.0774
4...	"	A	4	.0728	56..	6156	"	3	.0774
5...	3601	"	5	.0717	57..	6226	A	4	.0692
6...	3860	D	1	.0708	58..	6236	"	5	.0678
7...	3950	A	5	.0730	59..	6323	D	4	.0773
8...	3955	"	4	.0751	60..	6383	A	5	.0683
9...	3960	"	5	.0737	61..	6388	"	5	.0678
10...	3965	"	4	.0724	62..	6403	"	3	.0647
11...	3975	"	6	.0754	63..	6408	"	5	.0648
12...	3980	"	5	.0734	64..	6520	D	4	.0795
13...	4067	D	5	.0701	65..	6682	"	4	.0768
14...	4092	A	5	.0735	66..	6697	"	3	.0796
15...	4097	"	7	.0727	67..	6844	"	4	.0827
16...	4189	D	7	.0790	68..	6869	"	4	.0761
17...	4224	"	7	.0758	69..	6874	"	4	.0766
18...	4583	"	3	.0727	70..	6879	"	3	.0793
19...	4588	"	4	.0728	71..	6904	A	3	.0690
20...	4653	A	5	.0719	72..	6909	"	4	.0679
21...	4658	"	2	.0744	73..	7046	D	4	.0774
22...	4663	"	4	.0716	74..	7198	"	4	.0781
23...	4673	"	4	.0720	75..	7208	"	4	.0750
24...	4688	"	5	.0717	76..	7223	"	5	.0772
25...	4947	D	5	.0769	77..	7238	"	3	.0790
26...	5012	A	5	.0717	78..	7258	A	4	.0678
27...	5017	"	5	.0714	79..	7552	D	4	.0753
28...	5114	D	5	.0739	80..	7587	"	4	.0780
29...	5124	"	4	.0708	81..	7632	A	5	.0702
30...	5169	A	5	.0723	82..	7642	"	4	.0702
31...	5271	D	1	.0764	83..	7734	D	4	.0751
32...	5276	"	5	.0715	84..	7739	"	4	.0762
33...	5281	"	6	.0709	85..	7769	A	4	.0700
34...	5316	"	5	.0744	86..	7784	"	4	.0711
35...	5326	"	5	.0734	87..	7794	"	4	.0708
36...	5336	A	5	.0712	88..	7921	D	6	.0677
37...	5361	"	5	.0716	89..	7931	"	5	.0743
38...	5366	"	3	.0705	90..	7936	"	4	.0733
39...	5376	"	3	.0718	91..	7961	A	5	.0708
40...	5513	"	5	.0713	92..	7966	"	5	.0711
41...	5528	"	3	.0716	93..	8290	D	4	.0749
42...	5655	D	5	.0754	94..	8325	A	6	.0771
43...	5665	"	4	.0749	95..	8335	"	6	.0721
44...	5680	A	4	.0688	96..	8442	D	5	.0712
45...	5695	"	5	.0695	97..	8452	"	4	.0676
46...	5710	"	4	.0701	98..	8497	A	4	.0792
47...	5725	"	4	.0724	99..	8609	D	4	.0652
48...	5740	"	5	.0699	100..	8614	"	4	.0643
49...	5812	D	6	.0744	101..	8679	A	5	.0838
50...	5872	A	4	.0704	102..	8684	"	5	.0835
51...	6004	D	5	.0733	103..	9033	"	3	0.0890
52...	6039	A	5	0.0665					

obvious relation to the variation in the length of the period. The table contains nightly groups in order of Julian Day of all observations made by Wendell during maximum light and during the interval of totality at primary eclipse.

TABLE II

No.	Mean Julian Day	Branch	Number		Mean Phase of Crossing $8^{M}40$	O−C (Wendell)
			Obs.	Epochs		
1.....	2413552	D	7	2	$0^{d}0734$	$-0^{d}0009$
2.....	3577	A	12	3	.0718	− 7
3.....	4003	"	41	8	.0736	+ 11
4.....	4154	D	20	4	.0752	− 27
5.....	4586	"	7	2	.0728	− 3
6.....	4668	A	20	5	.0721	− 4
7.....	5066	"	15	3	.0718	− 7
8.....	5071	D	15	4	.0742	− 17
9.....	5299	"	21	4	.0725	0
10.....	5411	A	24	6	.0713	− 12
11.....	5720	D	15	3	.0749	− 24
12.....	5736	A	26	6	.0701	− 24
13.....	6102	D	16	4	.0766	− 41
14.....	6130	A	18	4	.0678	− 47
15.....	6395	"	18	4	.0666	− 59
16.....	6546	D	15	4	.0782	− 57
17.....	6866	"	15	4	.0786	− 61
18.....	7035	A	11	3	.0682	− 43
19.....	7182	D	20	5	.0772	− 47
20.....	7653	"	16	4	.0762	− 37
21.....	7720	A	21	5	.0704	− 21
22.....	7928	D	15	3	.0714	+ 9
23.....	8163	A	22	4	.0729	+ 4
24.....	8398	D	13	3	.0712	+ 13
25.....	8612	"	8	2	.0648	+ 77
26.....	8629	A	14	3	.0824	+ 99
27.....	9033	"	3	1	0.0890	+0.0165

The earlier work on the light-elements by Chandler was based upon more than three thousand observations by Knott, Wilsing, Yendell, Sperra, and Chandler, obtained during an interval of twenty-two years. The material has been adapted in Table IV so as to show the deviations from Wendell's light-elements. The observed times of minimum in the third column are derived by applying the residuals of the fourth column, taken from Chandler,[1] to the times predicted by means of his formula:

$$\text{Min.} = \text{J.D. } 2407890.3229 + 2^{d}4928761 \cdot E.$$

[1] *Op. cit.*

TABLE III

Maximum and Minimum Light

Julian Day	Mean Phase	Mean Max. Light	No. Obs.	Julian Day	Mean Min. Light	No. Obs.
2413491	2^d24	6^M76	2	2413531	9^M18	7
3589	0.53	6.81	5	3581	9.27	5
3626	0.24	6.78	2	3586	9.20	4
3705	1.92	6.74	5	3601	9.21	6
3734	1.00	6.76	2	3880	9.24	7
3913	0.44	6.78	5	3950	9.18	5
4741	0.83	6.80	12	3955	9.22	5
6430	2.09	6.82	2	4067	9.14	4
6431	0.60	6.78	3	4087	9.13	10
9405	0.60	6.87	5	4092	9.11	10
9415	0.62	6.86	2	4583	9.18	1
9429	2.15	6.94	4	4653	9.15	1
9436	1.70	6.83	4	5827	9.09	1
9441	2.11	6.88	2	6388	9.12	7
				6418	9.14	1
				6423	9.07	4
				6428	9.12	2
				6904	9.12	1
				7258	9.02	1
				8335	9.03	1
				8679	9.04	3
				8684	9.13	1
				9018	9.25	1
				9028	9.13	2

TABLE IV

Normals from Chandler

Number of Minima	Wt.	Mean Observed Time of Minimum	O−C (Chandler)	O−C (Wendell)
3	2	2408027^d4315	$+0^d0004$	$+0^d0222$
5	6	08194.4547	+ 9	+ 222
4	6	08386.4071	+ 19	+ 225
6	9	08533.4841	− 8	+ 193
4	4.5	08730.4204	− 17	+ 178
5	8	08887.4709	− 24	+ 166
5	9.5	09064.4640	− 35	+ 149
4	5	09251.4291	− 42	+ 137
2	3	09615.3926	− 6	+ 162
2	4	10313.3934	− 51	+ 95
7	10.5	10597.5931	+ 68	+ 204
3	5	10729.7082	− 6	+ 126
1	2	10921.6664	+ 62	+ 188
2	1	11664.5422	+ 49	+ 151
7	7	12676.6481	+ 31	+ 101
5	9	13167.7398	− 18	+ 37
3	4	13259.9756	− 24	+ 28
1	1	13693.7307	− 78	− 40
5	5	14922.7285	+ 21	+ 20
1	1	15326.5774	+ 51	+ 37
4	3.5	15982.1954	− 33	− 68

The few published observations of minima (complete enough to be of definite value) since the close of Wendell's series are summarized in Table V.[1] Though some are not of high weight, they all tend to confirm Wendell's last observations in showing a large deviation from his linear formula. The fourteen determinations are grouped into normals at the bottom of the table.

TABLE V

LATER OBSERVATIONS

No.	Relative Weight	Observed Time of Minimum	O−C (Wendell)	Observer
1	4	2418886.428	+0d.016	Bemporad*
2	1	9063.422	.016	"
3	4	9235.440	.024	"
4	3	9240.424	.022	"
5	3	9250.394	.021	"
6	.....	9417.417	.020	Lehnert†
7	.....	9422.399	.017	"
8	.....	9432.374	.020	"
9	.....	9442.346	.021	"
10	low	9554.518	.013	"
11	.....	9579.446	.013	"
12	5	9711.575	.019	Ginori‡
13	5	9933.443	.020	Bemporad§
14	4	2420115.423	.019	"
		Mean Julian Day		
1, 2	5	2418921	+0d.016	Bemporad
3, 4, 5	10	9241	.022	"
6, 7, 8, 9	.....	9428	.020	Lehnert
10, 11	.....	9571	.013	"
12	5	9711	.019	Ginori
13, 14	9	2420014	.020	Bemporad

* *Atti della Accademia Gioenia di scienze naturali in Catania*, Serie V, **5**, pp. 64–68, 1912; *ibid.*, **8**, pp. 5, 16, 17, 1915.

† *Astronomische Nachrichten*, **192**, 199, 1912; **194**, 165, 1913.

‡ *Rivista di Astronomia*, **7**, 245, 1913.

§ *Astronomische Nachrichten*, **199**, 217, 1914.

The normal residuals in Tables II, IV, and V are plotted in Fig. 2, with abcissae in Julian Days and ordinates giving the deviation from Wendell's formula;[2] the dotted line represents the

[1] All times in this paper are heliocentric and refer to the Greenwich meridian.

[2] The plotted square at Julian Day 2408129 is derived from a minimum very carefully observed at Harvard in 1881 by Pickering, Wendell, and Searle (*Harvard Annals*, **46**, 207, 1904).

deviation of Chandler's light-elements. This diagram, which completely illustrates the results of the present study, gives a history of the variations in the period of U Cephei for thirty-three years. Obviously neither Chandler's nor Wendell's elements suffice at present, but the former, with the addition of a sine term of a few

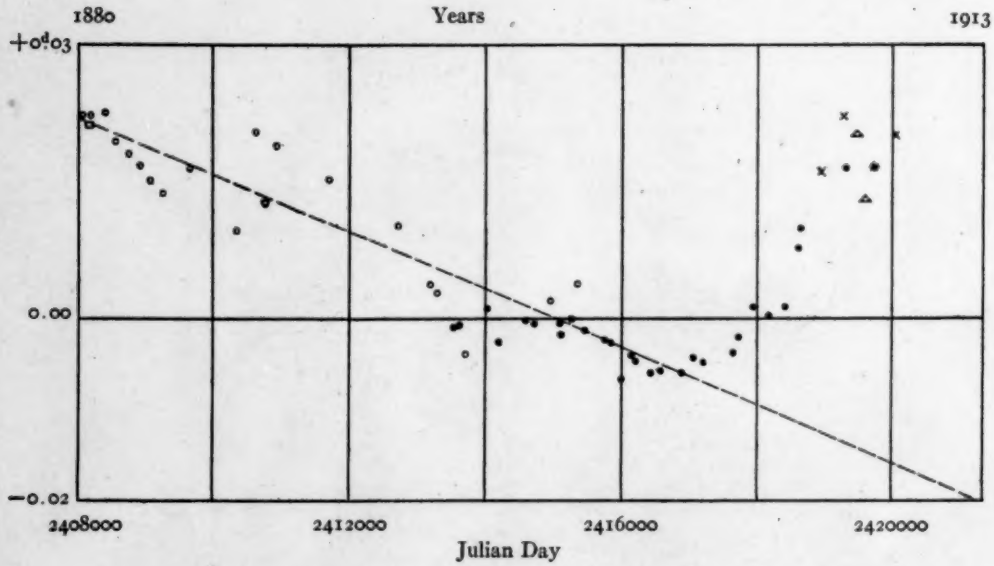


FIG. 2.—Deviations of the period of U Cephei from Wendell's linear elements

○ = Chandler
● = Wendell
× = Bemporad
△ = Lehnert
✻ = Ginori
□ = Pickering-Wendell-Searle

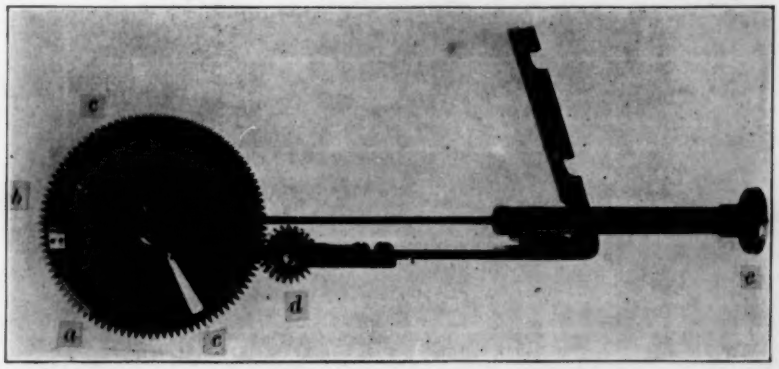
minutes' amplitude and twelve years' period, fulfils requirements up to 1900, though failing since that time. Wendell's elements serve best as a working formula at present, but would probably predict minima too early. The variations are apparently very complex and no attempt is made to obtain an analytical expression for them.

PASADENA, CAL.
April 3, 1916

Minor Contributions and Notes

A NEW DOUBLE OCCULTING SECTOR FOR STELLAR PHOTOGRAPHS

Various devices have been used to reduce the photographic image of a bright star, the parallax of which is sought, to near equality with the images of the comparison stars. The adjustable rotating sector of the type used by Schlesinger in his work at the Yerkes Observatory is commonly employed. Since it is almost always necessary to use comparison stars at least as faint as the



ninth or tenth magnitude, even the maximum reduction, about five magnitudes, makes it impossible adequately to reduce the image of a parallax star brighter than magnitude 3.5 or 4.0.

A simple form of double sector was recently suggested by Mr. Frank R. Sullivan, engineer in charge of the 40-inch refractor, and it has now been constructed in our shops. The photograph reproduced herewith shows the essential parts of the apparatus. It consists of two toothed disks of the same diameter (74 mm), mounted on the same shaft and separated by about 2.5 mm. The second disk, visible through the openings in the main sector at *cc*, has 99 teeth and runs freely on the axis. The main sector *b* has 100 teeth

and clamps down tight on a shoulder by means of a winged nut. A beveled gear behind the disks transmits the motion to the main sector. Both disks mesh with the free pinion *d*, with the result that the second "creeps" one revolution in every hundred. The opening in the second sector is 6°, while each of the two opposite openings *cc* in the main sector can be varied from 0° to nearly 90°. The beveled gear *e* engages a small pinion on the motor shaft.

Assuming 2.75 as the exposure factor for a difference of one stellar magnitude, the reduction of light with this double sector in magnitudes is $2.28 \log \frac{36000}{2n^\circ}$ if both openings in the main sector are left open. For the greatest reduction one of the openings in the main sector may be covered and the denominator in the foregoing expression becomes simply n. Table I gives directly the reduction in magnitudes for scale settings (read at *b*) on the main sector.

TABLE I

Scale Divisions	Degrees Open	Reductions For	
		Single Opening	Double Opening
1.....	2	9.7	9.0
2.....	4	9.0	8.3
3.....	6	8.6	7.9
4.....	8	.....	7.6
5.....	10	.....	7.4
10.....	20	.....	6.7
20.....	40	.....	6.0
40.....	80	.....	5.4

Our usual speed for this sector is five revolutions per second. If the main sector is opened three divisions or 6°, its other sector being covered, we shall get in a 10-minute exposure a total exposure for the occulted star of $\frac{1}{10}$ second made up of at least 30 uniformly distributed exposures. The double sector is interchanged with the usual single sector in a few seconds by removing the one and clamping in the other. All the stars north of declination -15° as bright as magnitude 2.0 have been placed on the program for a determination of parallax with the 40-inch refractor. A plate of the field of

Sirius recently obtained with this sector shows Sirius as a star of magnitude 8.5 on the *B.D.* scale and having a round image suitable for measurement with five minutes' exposure. Another plate of the field of Procyon shows this bright star like one of *B.D.* magnitude 9.1. The five-minute exposure gave an almost ideal image for measurement.

OLIVER J. LEE

YERKES OBSERVATORY
April 18, 1916

Reviews

The Solar Rotation in June 1911. By J. B. Hubrecht. "Annals of the Solar Physics Observatory, Cambridge," Vol. III, Part I. Cambridge: University Press, 1915. 4to, pp. 77. 9 *s.*

The investigation of the rotation period of the sun by spectroscopic methods has received an exceptionally large amount of attention from astrophysicists during the past ten years. The remarkable character of the law of the sun's rotation and the possibilities of variation in rotation period and in the behavior of different elements have combined to add to the attraction which a radial velocity investigation of high precision naturally possesses for astronomers.

In this volume of the "Annals of the Solar Physics Observatory of Cambridge" are contained the results of an investigation of the solar rotation by Dr. Hubrecht in June, 1911. The instruments used were the McClean horizontal telescope, consisting of a coelostat and 12-inch objective in conjunction with a Littrow grating spectrograph of 14 feet focal length. The spectrograph may be rotated about a horizontal axis in such a way as to secure any desired position angle, and the sun's image is moved with reference to the slit by slow-motion controls on the auxiliary plane mirror. The exposures upon each photograph were made successively, the spectrograph remaining unchanged in position and the image of the sun being moved to bring upon the slit the points between which it was desired to institute a comparison. The most interesting feature of Dr. Hubrecht's observations is the fact that these comparisons were made between points distant from one another by 90° of latitude instead of 180°, as in investigations by previous observers. This has made it possible to discuss the question of the symmetry of the rotation law in the two solar hemispheres.

A description of the method of measurement and reduction of the photographs and the detailed results for the individual plates follow the account of the method of observation. In this connection the criticism should perhaps be made that no details are given regarding the color correction of the image-forming objective. If the focus of the solar image upon the spectrograph slit is made for visual light the color

curve of the objective must be of quite an exceptional character in order that the diameter of the image in light of wave-length 4300 be changed by but one part in one thousand.

From a discussion of his results Dr. Hubrecht arrives at the important conclusion that there is a variation of velocity with wave-length amounting to about 3 per cent at a maximum in the interval between λ 4300 and λ 4400. This difference, after an extended discussion of other possible causes, the author concludes to be due to ray curving in the solar atmosphere, a result which will probably be accepted with some question by astronomers. Naturally the chief objection to this explanation is that if carried to its logical conclusion the difference between the values obtained in the violet and the yellow and red portions of the spectrum should be very large, much larger than there is as yet any reason to suppose.

The result of a comparison of individual lines and elements indicates no certain differences of velocity for different elements, a conclusion in agreement with that of other observers except those at Mount Wilson and Marseilles. Attention may, however, be called to the fact that the enhanced lines should be distinguished from the arc lines in such a comparison. Thus for titanium we have as a percentage residual in the table given:

3 arc lines+0.3 3 enhanced lines−0.3

These quantities are small and may possess no significance. They are, however, in the direction found by some other observers.

The lack of symmetry found by Dr. Hubrecht between the Northern and Southern hemispheres as regards rotational values is a result of great interest, and provides ample justification for the method of observation adopted by him. The existence of local drifts during the comparatively short interval of time covered by the observations is a most probable explanation of the differences obtained. This may perhaps account as well for at least a part of the discrepancy between his result of 13°.2 for the angular velocity at the equator and that of Plaskett of 14°.4 for a slightly later epoch. In view of the probability of a variation in the sun's period of rotation, as shown by recent observations, such a comparison should be limited to results of closely the same date.

Although some of the results obtained from this investigation of Dr. Hubrecht must await confirmation from future observations, the questions raised will add materially to the interest already attaching to the study of this important problem.

W. S. ADAMS

KARL SCHWARZSCHILD

It is with deepest regret that we record the death, on May 11 at Potsdam, of Karl Schwarzschild, director of the Royal Astrophysical Observatory of Potsdam. His name has been associated with the *Journal* as a collaborator since 1911. In the death of Schwarzschild, science has suffered an irreparable loss. Hardly more than forty-two years of age, he had already distinguished himself by his contributions to mathematics, physics, and to both practical and theoretical astronomy. We hope to publish in a future issue an appropriate sketch of his life and work.